



## Concepts and Functions in the Building Engineering



Journal homepage: <https://cfbejournal.abu.ac.ir/>

---

### Optimal design method of retaining wall using the Intelligent Water Drops (IWD) metaheuristic algorithm

---

Mehdi Shalchi Tousi   DOI: [10.22034/IJSCEBG.2024.194526](https://doi.org/10.22034/IJSCEBG.2024.194526)

Assistant Professor, Faculty of Engineering, Ahlul Bayt (a.s.) International University.

\* **Corresponding author:** [mehdishalchi@abu.ac.ir](mailto:mehdishalchi@abu.ac.ir)

---

#### ARTICLE INFO

Article history:

Received: 11 June 2024

Revised: 26 July 2024

Accepted: 02 September 2024

---

#### Keywords:

Retaining wall, Algorithm, Smart water droplets, Rebar.

#### ABSTRACT

Water drops in rivers can intelligently find the shortest path to the sea. The Intelligent Water Drops (IWD) algorithm was introduced in 2007 based on this behavior. In this algorithm, water drops have two important features: velocity and the amount of soil they carry, which they collect from the ground. The less soil they have, the faster they can move. The soil amount represents information exchanged between the ground and the water drops; as more drops pass over a piece of ground, its soil amount decreases. A drop prefers a path with less soil. If the movement steps of the drops are considered discrete, it can be assumed that the drop moves on the nodes of a graph. In the algorithm, water drops represent candidate solutions and are randomly initialized at the nodes of the graph.

E-ISSN: 000-000

© 2025 The Authors. Concept and Function in the Building Engineering by Ahlul Bayt International University.

How to cite this article:

M. Shalchi Tousi, (2023). Optimal design method of retaining wall using the Intelligent Water Drops (IWD) metaheuristic algorithm, 1(1), 1-19. <https://doi.org/10.22034/IJSCEBG.2025.194526>

## **Introduction**

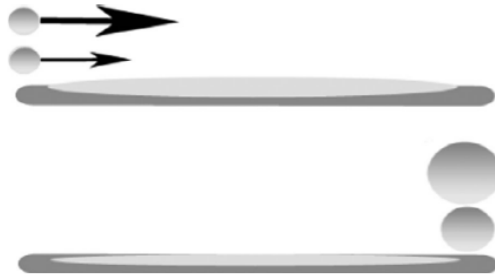
**Introduction** In this research, the Intelligent Water Drops Algorithm is used to optimize retaining walls. This algorithm, introduced by Shah-Hosseini in 2009, is a novel optimization algorithm based on swarm intelligence, inspired by the observation of water drops flowing in a river [1]. The nature of a river enables it to select the optimal path among multiple routes, from the source to the destination. These optimal paths result from the interactions and reactions between the water drops themselves and between the water drops and the riverbed. First, the algorithm is fully introduced and explained. Then, by defining a cantilever concrete retaining wall, the design variables—corresponding to the unknowns of the problem—and the design constraints, based on design standards in accordance with ACI codes and geotechnical failure criteria, are formulated.

## **Foundation of the Intelligent Water Drops Algorithm**

**The Basis of the Intelligent Water Drops Algorithm** The gravitational force acting on water droplets in a river causes them to move and flow towards their destination. In the absence of obstacles, water droplets choose the direct path, which is the shortest route from origin to destination. However, the presence of various obstacles makes the actual, winding path different from the ideal one. Nevertheless, this path appears to be the optimal route between origin and destination. Each intelligent water drop (IWD) has two main characteristics: \* Speed; \* The amount of soil it can carry. As shown in Figure (1), a water drop can carry some soil while moving in the river from one point to another. In fact, some of the soil in the riverbed can be displaced by the water drop and added to the soil carried by each drop. This means that by moving to the point on the right (Figure 1), the amount of soil accumulated by the water drop increases, and the soil in the path decreases. Also, speed plays an important role in displacing the riverbed soil. In Figure (2), two water drops with the same amount of carried soil and two different speeds are shown. The water drop with the larger arrow indicates a higher speed. The drop with the higher speed accumulates more soil at the end than the other. In other words, the soil-carrying characteristic of the drop depends on its speed; if the speed of the drop increases, it carries more soil. The speed of the IWD is higher on a path with less soil than on a path with significant soil. In Figure (3), two identical drops with the same speed are shown on two different paths. The water drop has a higher speed in the path with less soil and can carry more soil. However, the path with more soil has more resistance to the flow, and the water drop accumulates less soil at a lower speed. This means that the speed of each IWD in moving from the current position to the next position increases in inverse proportion to the amount of soil between the two positions. Therefore, an IWD will have a higher speed on a path with less soil than on a path with more soil.



**Figure 1.** Soil transported by a water droplet



**Figure 1.** Water droplet velocity and associated soil erosion.



**Figure 3.** Comparison of the performance of two pavements with different subgrade soils

The two mentioned properties, velocity and the amount of soil carried by the drop, may change along a flow from source to destination. Each IWD starts its path with an initial velocity and zero carried soil amount. Then, by moving through the environment, it accumulates some soil and gains some velocity. Each IWD requires a path selection mechanism to determine the next position. In this mechanism, the IWD prefers paths with less soil over those with more soil. This is achieved by imposing a uniform random distribution on the soil amounts present on the paths. The selection of the next path is inversely proportional to the amount of soil on the paths. Therefore, the path with less soil has a higher chance of being selected by the IWD.(11)

### Intelligent Water Drops (IWD) Algorithm

In the IWD algorithm, the problem is represented as a graph (N, E), where N is the set of nodes forming the path and E is the set of paths through which the water drops move. The solution process begins by traversing the nodes along the path and continues until the IWD finds the minimum solution value for the problem. After each iteration, the best solution found in that iteration—referred to as  $T^{IB}$ —is obtained. To achieve the overall best solution,  $T^{TB}$ , the best solution from each iteration must be utilized.

The algorithm then initiates another iteration with new IWDs and the same soil distribution on the graph's paths, repeating the aforementioned process. The algorithm terminates when it reaches either the maximum number of iterations ( $iter\_max$ ) or the lowest global solution with the desired quality.

There are two types of parameters in the IWD algorithm: Static Parameters: Remain constant throughout the entire algorithm execution. Dynamic Parameters: Reinitialized after each iteration. The step-by-step procedure of the proposed algorithm is elaborated further [11].

#### 1) Initialization of Static Parameters:

The problem is input to the algorithm as a graph (N, E). Initially, the quality of the best total solution  $T^{TB}$  is set to its worst possible value:

$$q(T^{TB}) = +\infty$$

The maximum number of iterations  $iter_{max}$  is determined by the user based on the desired solution accuracy and algorithm runtime. The iteration counter  $iter_{count}$  is set to zero at the start of the algorithm. The number of water drops  $N_{IWD}$  is a positive integer, typically set equal to the number of nodes in the graph.

Fixed Parameters Required for Velocity and Soil Updates:

- For velocity update:

$$c_v = 1 \quad \text{and} \quad b_v = 0.01 \quad a_v = 1$$

- For soil update:

$$c_s = 1 \quad \text{and} \quad b_s = 0.01 \quad a_s = 1$$

- For local soil update:<sup>1</sup>

$$\rho_n = 0.9$$

---

1. The local soil

- For global soil update:  $\rho$

$$\rho_{IWD} = 0.9$$

The *InitSoil* value on each path is denoted by the constant *InitSoil*, meaning the soil between any two nodes  $(i)$  and  $(j)$  is expressed as:  $soil(i, j) = InitSoil$ . Additionally, the initial velocity of each IWD is set as *InitVel*. Both parameters are user-defined. In this study, based on the reference paper, the values 10,000 and 200 are adopted for *InitSoil* and *InitVel*, respectively, which can be adjusted according to the problem's requirements.

**2) Initialization of Dynamic Parameters:** As mentioned earlier, dynamic parameters change with each algorithm iteration. The list of visited nodes  $V_c(IWD)$  also falls into this category and is initially empty:

$V_c(IWD) = \{ \}$  After each iteration, the visited nodes are added to this list.

3. IWDs are randomly distributed across the graph nodes as their first observed nodes.

4. The visited node list for each IWD is updated.

5. Steps 1-5 to 4-5 are executed for each IWD iteration.

1-5) For an IWD located at node  $i$ , the next node  $j$  is selected using the following probability function, ensuring no problem constraints are violated and  $j$  is not in the visited node list.

$$p_i^{IWD}(j) = \frac{f(soil(i,j))}{\sum_{k \in V_c(IWD)} f(soil(i,k))} \quad (1-3)$$

where:

$$f(soil(i,j)) = \frac{1}{\varepsilon_s + g(soil(i,j))} \quad (2-3)$$

$$g(soil(i,j)) = \begin{cases} soil(i,j) & \text{if } \min_{l \in V_c(IWD)}(soil(i,l)) \geq 0 \\ soil(i,j) - \min_{l \in V_c(IWD)}(soil(i,l)) & \text{else} \end{cases} \quad (3-3)$$

Here's the precise academic English translation of your text:

The newly selected node  $j$  is added to the visited nodes list  $V_c(IWD)$

2-5) For each IWD moving from node  $i$  to  $j$ , the velocity is updated according to  $vel^{IWD}(t)$

$$vel^{IWD}(t+1) = vel^{IWD}(t) + \frac{a_v}{b_v + c_v \cdot soil^2(i,j)} \quad (3-4)$$

where  $vel^{IWD}(t + 1)$  represents the updated velocity at the new node  $j$ .

3) Soil Deposit Calculation (Step 5-3):

5-3)The soil accumulated during movement from  $i$  to  $j$  is computed by:  $\Delta soil(i, j)$

$$\Delta soil(i, j) = \frac{a_s}{b_s + c_s \cdot time^2(i, j; vel^{IWD}(t+1))} \quad (3-6)$$

$$time(i, j; vel^{IWD}(t + 1)) = \frac{HUD(j)}{vel^{IWD}(t+1)} \quad (3-7)$$

where the traversal time is:

$HUD(j)$  denotes a heuristic value defined by the user according to problem specifications.

( 4-5) Both the path soil and carried soil  $soil(i, j)$  quantities are  $soil^{IWD}$  updated through:

$$soil(i, j) = (1 - \rho_n) \cdot soil(i, j) - \rho_n \cdot \Delta soil(i, j) \quad (3-8)$$

$$soil^{IWD} = soil^{IWD} + \Delta soil(i, j) \quad (3-9)$$

6) The best iteration solution  $T^{IB}$  is derived from all candidate solutions  $T^{IWD}$  obtained by the IWDs through the following selection formula:

$$T^{IB} = \arg \max_{T^{IWD}} q(T^{IWD}) \quad (3-10)$$

where the function  $q(T^{IWD})$  represents the quality of the solution.

The soils on the paths of the best solution in iteration  $T^{IB}$  are updated using the following formula:

$$soil(i, j) = (1 + \rho_{IWD}) \cdot soil(i, j) \quad (3-11)$$

$$-\rho_{IWD} \cdot \frac{1}{(N_{IB}-1)} \cdot soil_{IB}^{IWD} \quad \forall (i, j) \in T^{IB} \quad (3-12)$$

where  $N_{IB}$  is the number of nodes  $T^{IB}$  in the solution .

8) The global best solution  $T^{TB}$  is updated by the iteration's best solution as follows

$$T^{TB} = \begin{cases} T^{LB} & \text{if } q(T^{TB}) > q(T^{IB}) \\ T^{TB} & \text{otherwise} \end{cases} \quad (3-13)$$



**Table: 1-3:** Problem variables and software output for retaining wall design

Type	Symbol	One	Name	Categorization
Constant	$B$	Metre ( $m$ )	Base width	Geometric specifications of the wall
	$B_{to}$		Width of the paw	
	$B_s$		Thickness of the end of the stem	
	$D_b$		Thickness of the wall foundation	
	$t_t$		Thickness of the shoot tip	
Constant	$AstS$	Square meters per one meter of wall length ( $m^2/m$ )	Tensile steel cross-section of the leg	Specifications of Consumable Steels
	$AstT$		Cross-sectional area of tensile steel in the anchorage	
	$AstH$		Tensile steel cross-section of the heel	
	$AscS$		Cross-sectional area of the compressive steel in the column	
	$AscT$		Cross-sectional area of the compression steel of the anchorage	
Discrete	$AscH$	Megapascal ( $MPa$ )	Cross-sectional area of the heel compression steel	Specifications of Consumable Steels
	$Fy1$		Tensile yield strength of shaft steel	
	$Fy2$		Compressive yield stress of the shaft steel	
	$Fy3$		Tensile yield stress of claw steel	
	$Fy4$		Yield stress of compressive steel claw	
	$Fy5$		Yield stress of tensile heel steel	
Discrete	$Fy6$	Millimeter ( $mm$ )	Compressive yield stress of heel steel	Specifications of Consumable Steels
	$d_b$		Rebar Diameter	

Discrete	$F_{cs}$	Megapascal (MPa)	Compressive strength of shaft concrete	Concrete Specifications
	$F_{cf}$		Compressive strength of foundation concrete" or "Compressive strength of concrete in the foundation	
Discrete	$n_1$	-	Number of tensile reinforcement bars in the stem	Software output
	$n_2$		Number of tensile reinforcement bars in the heel	
	$n_3$		The number of tensile reinforcement bars in the heel	
	$n_4$		Number of compression reinforcement in the stem	
	$n_5$		Number of compression reinforcements in the claw	
	$n_6$		Number of compression reinforcements in the heel	

Here is a precise English translation of your text:

As indicated in the table above, some variables are continuous, while others are discrete.

It is evident that all continuous variables have upper and lower limits.

### 3-3-2 Objective Functions

Two objective functions—**weight** and **cost**—are introduced. The goal of optimization is to determine the dimensions and specifications of the wall while adhering to structural and geotechnical constraints, such that both the weight and cost of the wall are minimized. The objective functions used in this study are the same as those in the Saribas and Erbatur paper and are defined as follows:

know if you'd like any refinements

$$f(C) = C_s W_s + C_c V_c \quad (3-16)$$

$C_s$  = Unit cost of steel (\*\*\\$/kg\*\*) ( $\$/kg$ ) = Unit cost of concrete (covering formwork, pouring, vibration, and labor expenses) (\*\*\\$/m<sup>3</sup>\*\*) (\*\*\\$/m<sup>3</sup>\*\*) )

$W_{st}$  = Weight of steel per unit length of the wall (\*\*kg\*\*) (\*\*kg\*\*)

$(V_c)$  = Volume of concrete per unit length of the wall (\*\*m<sup>3</sup>\*\*)

Additionally, based on the recommendations of Saribas and Erbatur, the coefficients  $(C_s)$  and  $(C_c)$  remain constant throughout the optimization process, with values of 0.4 and 40,

$$f(W) = W_s + 100V_c\gamma_c \quad (17)$$

Where:

-  $\gamma_c$  = Unit weight of concrete (kN/m<sup>3</sup>)

To calculate the weight of steel used, the cross-sectional area of each part must be multiplied by its length used in the wall

In tension

$$l_{dh} = \frac{0.02F_y d_b}{\sqrt{f_c}} \quad (18-3)$$

$$l_{dc} = \max\left(\frac{0.02F_y d_b}{\sqrt{f_c}}, 0.0003F_y d_b\right) \quad (19-3)$$

Through these formulas, the basic development length is obtained, which must then be multiplied by the applicable modification factors. The modification factors relevant to the conditions of this study are as follows:

1) When the provided reinforcement exceeds the required reinforcement, the basic development length shall be multiplied by the following ratio. This factor applies to both tension and compression cases

$$\lambda_1 = \frac{A_s \text{ provided}}{A_s \text{ required}} \quad (20-3)$$

2) For No. 11 bars and smaller (No. 11 bar diameter is 1.41 inches or 35.81 mm) with concrete cover not less than 0.5-2 inches, and for 90-degree hooks with minimum 2 inches of concrete cover behind the hook, the modification factor is 0.7. This provision applies only to tension reinforcement.

Therefore, the total required development length is calculated as follows:

$$L_{dh} = \lambda_1 \lambda_2 l_{dh} \quad (21-3)$$

$$L_{dc} = \lambda_1 l_{dc} \quad (22-3)$$

"After performing the above calculations, the 90-degree hook length (as specified by the code .  
".as 12d<sub>b</sub>) must be added to the obtained values

$$W_{stem} = A_{stem} \times (H_s + L_{dh\ or\ dc} + 12d_b - cover) \times 7850 \quad (23-3)$$

$$W_{heel} = A_{heel} \times (B_{heel} + L_{dh\ or\ dc} + 12d_b - cover) \times 7850 \quad (24-3)$$

$$W_{toe} = A_{toe} \times (B_{toe} + L_{dh\ or\ dc} + 12d_b - cover) \times 7850 \quad (25-3)$$

. "In the above formulas:

- W = steel weight (kg)
- A<sub>s</sub> = cross-sectional area (m<sup>2</sup>)
- γ<sub>steel</sub> = unit weight of steel (kg/m<sup>3</sup>)

"L = lengths (m) -

### 3-3-3 Design Constraints

In essence, the philosophy of design is to fulfill requirements based on structural behavior. In the design process of a retaining wall, these requirements represent potential structural and geotechnical failure modes. Therefore, these requirements—which govern the design variables—are referred to as constraints.

In programming, constraints are denoted by the symbol g and are defined as inequality equations :

$$g_i(x) \leq 0, \quad i = 1, 2, \dots, m \quad (26-3)$$

1) Geotechnical Constraints [13,14]

- **Overturning Stability Check:** The wall must be stable against overturning. According to standard references, this stability is evaluated by the ratio of resisting moment to overturning moment about the toe of the wall (Point O, Figure 3-5)

$$\frac{M_r}{M_o} \geq SF_o \rightarrow (M_o \times SF_o) - M_r \leq 0 \quad (27-3)$$

> "Where \$SF\_o\$ is the factor of safety against overturning, with a value ranging from 1.5 to 2. The moments \$M\_r\$ and \$M\_o\$ are calculated using Equations (3-28) and (3-29), respectively."

$$M_r = \sum W_i \bar{x} + P_{av} B \quad (28-3)$$

- **W<sub>i</sub>:** Forces due to the weight of the wall and the backfill soil (kN)

- $\bar{x}$ : Distance from the toe to the point of application of gravitational forces (m)
- $\bar{y}$ : Distance from the base level to the point of application of the active lateral forces (m)
- $P_{av}$ : Vertical component of the active force acting in a resisting manner (kN)

$$M_o = P_{ah}\bar{y} \quad (29-3)$$

$P_{ah}$ : Horizontal component of the driving force(kN) ;The resultant force due to lateral earth pressure and surcharge is evaluated based on either Rankine's or Coulomb's theory; in the present study, Rankine's theory has been adopted.

$$k_a = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \quad (30-3)$$

$$P_a = \left[ \frac{1}{2} H_T^2 \gamma_r k_a \right] + [q k_a H_T] \quad (31-3)$$

Where  $H_T$  is the total height of the backfill measured from the base level (m),  $\gamma_r$  (kN/m<sup>3</sup>) is the unit weight of the backfill soil (kN/m<sup>3</sup>), and  $q$  is the magnitude of the surcharge load (kPa). It is worth noting that the contribution of the embedded depth in front of the wall has been neglected in calculating the resisting force due to the following reasons.\*

1. Erosion and weathering
2. Water accumulation
3. Traffic loads
4. Urban excavations for utility installations
5. Uncertainty regarding sufficient wall displacement to mobilize resistance

Therefore, it can be written as follows:

$$P_{ah} = P_a \cos \beta \quad , \quad P_{av} = P_a \sin \beta \quad (32-3)$$

**Sliding Check:** The wall must be stable against sliding. Sufficient friction must exist between the base slab and the soil, which is expressed as follows. The factor of safety against sliding  $SF_s$  typically ranges from 1.25 to 2.

$$\frac{F_r + P_p}{P_{ah}} \geq SF_s \quad \rightarrow \quad (P_{ah} \times SF_s) - (F_r + P_p) \leq 0 \quad (33-3)$$

- $P_p$ : The passive resistance force in front of the wall, which, as previously explained, has been neglected.
- $F_r$ : Frictional force between the foundation and the base slab (kN)

$$F_r = R \times \mu + C_a \times B \quad (34-3)$$

$$\mu = \tan \beta \quad (35-3)$$

- $\mu$ : Coefficient of base friction
- $R$ : Resultant of all vertical forces ( $kN$ )
- $C_a$ : Base adhesion; its value ranges from  $0.6c$  to  $1c$ , and in this study, a value of  $0.7c$  is adopted.
- **Check for Tension in the Base:** To prevent the development of tension beneath the footing, the eccentricity ( $e$ ) must be less than  $\frac{B}{6}$ .

$$e \leq \frac{B}{6} \rightarrow e - \frac{B}{6} \leq 0 \quad (36-3)$$

$$M_{net} = M_r - M_o \rightarrow \bar{x} = \frac{M_{net}}{\Sigma V} \rightarrow e = \frac{B}{2} - \bar{x} \quad (37-3)$$

**Bearing Capacity Check:** In this study, Hansen's equations are used to calculate the ultimate bearing capacity. According to reference texts, the factor of safety for bearing capacity ranges from 2 to 4 for granular soils and from 3 to 5 for cohesive soils. However, it is worth noting that a value of 3 is commonly recommended in all standard references. This check is performed as follows:

$$\frac{q_{ult}}{q_{max}} \geq SF_b \rightarrow (q_{max} \times SF_b) - q_{ult} \leq 0 \quad (38-3)$$

$$q_{max} = \left(\frac{\Sigma V}{B}\right) \left(1 + \frac{6e}{B}\right), \quad q_{min} = \left(\frac{\Sigma V}{B}\right) \left(1 - \frac{6e}{B}\right) \quad (39-3)$$

$$q_{ult} = cN_c S_c d_c i_c g_c b_c + \bar{q} N_q S_q d_q i_q g_q b_q + 0.5 \gamma_b \dot{B} N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma \quad (40-3)$$

In the above equation,  $\gamma_b$  is the unit weight of the foundation soil ( $kN/m^3$ ) ( $kN/m^3$ ), which may be equal to the unit weight of the backfill. Due to the extensive volume of content, the explanation and definition of the parameters involved in the bearing capacity formula have been omitted here, and full details can be found in Reference [13]. Also,  $q_{ult}$ ,  $q_{max}$ , and  $q_{min}$  are expressed in kPa.

It should be noted that in the first example, which serves as a validation case based on the study by Saribash and Erbatur, the bearing capacity was calculated using the Meyerhof method. Therefore, this method has been applied only in that specific case. The bearing capacity in this method is calculated using the following equation:

$$q_{ult} = cN_c S_c d_c + \bar{q} N_q S_q d_q + 0.5 \gamma_b \dot{B} N_\gamma S_\gamma d_\gamma \quad (41-3)$$

## 2- Structural Requirements [References 12, 15]

**Shear Check:** The concrete dimensions and the characteristic strength of the concrete must be sufficient to resist the shear forces applied to all three sections: the stem, the toe, and the heel.

$$V_n \times \phi_V \geq V_u \rightarrow V_u - (V_n \times \phi_V) \leq 0 \quad (42-3)$$

Where  $V_u$  is the applied shear force on the section, expressed in  $kN$ . The critical sections for shear and bending checks are illustrated in Figure 3-3. The sections shown in the figure—(1), (2), and (3)—correspond to the stem, heel, and toe, respectively. It should be noted that shear checks for the stem are performed at heights of  $\frac{1}{4}H_s$ ,  $\frac{1}{2}H_s$ ,  $\frac{3}{4}H_s$  and  $H_s$ . The strength reduction factor for shear,  $\phi_V$ , is 0.75 as per the ACI code. Moreover, a load factor of 1.4 is considered in the calculation of  $V_u$ . The forces acting on the stem, heel, and toe are shown in Figure 3-7.

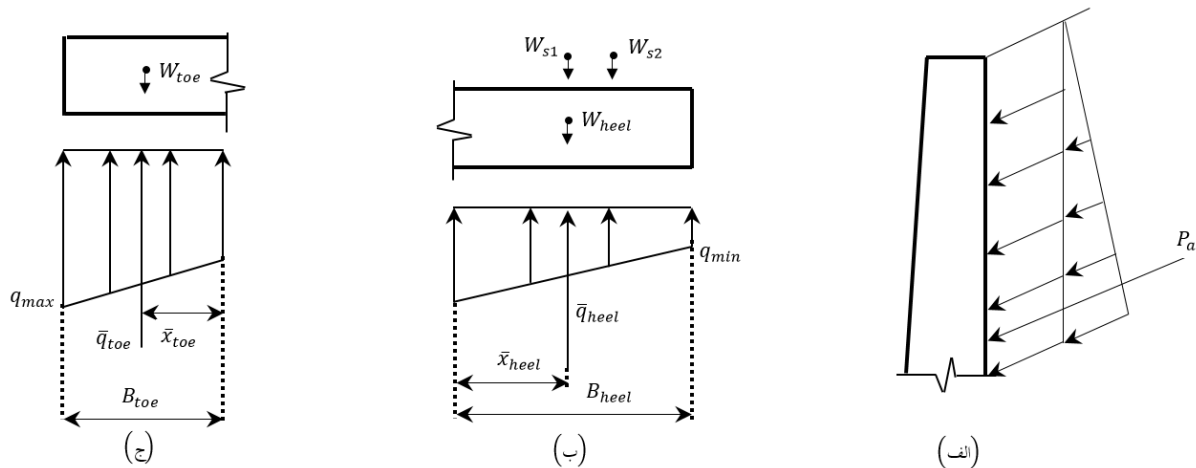
$$V_n = \frac{1}{6} \sqrt{f_c} d \quad (43-3)$$

$$V_{u_{stem}} = \cos \beta H_s k_a \left[ q + \left( \frac{1}{2} \gamma_r H_s \right) \right] \quad (44-3)$$

$$V_{u_{heel}} = W_{heel} + W_{s1} + W_{s2} + (q \cos \beta B_{heel}) - \bar{q}_{heel} \quad (45-3)$$

$$V_{u_{toe}} = \bar{q}_{toe} - W_{toe} \quad (46-3)$$

- $B_{heel}$ : Width of the heel ( $m$ )
- $\bar{q}_{heel}$ : Equivalent load applied to the base of the heel ( $kN$ )
- $\bar{q}_{toe}$ : Equivalent load applied to the base of the toe ( $kN$ )
- Additionally, the forces from each component, denoted by  $W$ , are expressed in kilonewtons ( $kN$ ).



**Figure 3-7:** Applied forces on (a) the stem, (b) the heel, and (c) the toe

**Flexural Check:** All sections of the wall must be capable of resisting the applied bending moments.

$$M_n \times \phi_M \geq M_u \quad \rightarrow \quad M_u - (M_n \times \phi_M) \leq 0 \quad (47-3)$$

where  $0.9 M\phi_M$ , the strength reduction factor for flexure, is 0.9. As previously mentioned, the design is such that if compressive reinforcement is not required, only the tensile reinforcement will govern.

### Tensile Reinforcement

$$a = \frac{A_s F_y}{0.85 f_c b} \quad (48-3)$$

$$M_n = A_s F_y \left( d - \frac{a}{2} \right) \quad (49-3)$$

### Tensile and Compressive Reinforcement

$$a = \frac{A_s F_y - A'_s F'_y}{0.85 f_c b} \quad (50-3)$$

$$M_n = 0.85 f_c a \left( d - \frac{a}{2} \right) + A'_s F'_y (d - \hat{d}) \quad (51-3)$$

Since the analysis is performed per one meter length of the wall, the value of LL in the above formulas is taken as

$$M_{u_{stem}} = \frac{1}{2} \cos \beta H_s^2 k_a \left[ q + \left( \frac{1}{3} \gamma_r H_s \right) \right] \quad (52-3)$$

$$M_{u_{heel}} = \left( \frac{1}{2} W_{heel} B_{heel} \right) + \left( \frac{1}{2} W_{s1} B_{heel} \right) + \left( \frac{2}{3} W_{s2} B_{heel} \right) + \left( \frac{1}{2} q \cos \beta B_{heel}^2 \right) - (\bar{q}_{heel} \bar{x}_{heel}) \quad (53-2)$$

$$M_{u_{toe}} = (\bar{q}_{toe} \bar{x}_{toe}) - \left( \frac{1}{2} W_{toe} B_{toe} \right) \quad (54-3)$$

In the above formulas, qq and WW are expressed in kilonewtons (kN).

**Minimum Tensile Reinforcement:** The cross-sectional area of tensile reinforcement must exceed the minimum value specified by the code. This requirement is checked for all three sections: the stem, the heel, and the toe.

If needed, I can adjust the text in accordance with the ACI or Eurocode standards. Would you like to refer to either of these codes?\*

$$A_s \geq A_{s_{min}} \quad \rightarrow \quad A_{s_{min}} - A_s \leq 0 \quad (55-3)$$

$$A_{s_{min}} = \max\left(\frac{0.25bd\sqrt{f_c}}{F_y}, \frac{1.4bd}{F_y}\right) \quad (56-3)$$

- **Yielding of Tensile Reinforcement:** The verification of this criterion depends on the presence or absence of compressive reinforcement.

$$\rho \leq \rho_b \quad \rightarrow \quad \rho - \rho_b \leq 0 \quad (57-3)$$

$$\rho_b = 0.85\beta_1 \frac{f_c}{f_y} \times \frac{600}{600+F_y} \quad (58-3)$$

### Tensile and Compressive Reinforcement

$$\rho \leq \bar{\rho}_b \quad \rightarrow \quad \rho - \bar{\rho}_b \leq 0 \quad (59-3)$$

$$\bar{\rho}_b = \rho_b + \rho' \frac{f'_{sb}}{F_y} \quad (60-3)$$

$$f'_{sb} = 600 - \frac{d}{a}(600 + F_y) \leq F_y \quad (61-3)$$

The value of  $\beta_1$  varies with the specified compressive strength  $f'_c$ . According to the code, for strengths between 17 MPa and 25 MPa,  $\beta_1$  is 0.85. For strengths greater than 28 MPa, the value of  $\beta_1$  is reduced by approximately 0.05 for every 7 MPa (or 1000 psi) increase in  $f'_c$ .

### Yielding of Compressive Reinforcement:

$$\rho \geq \bar{\rho}_{min} \quad \rightarrow \quad \bar{\rho}_{min} - \rho \leq 0 \quad (62-3)$$

$$\bar{\rho}_{min} = \rho' \frac{F_y}{f_s} + 0.85\beta_1 \frac{f_c}{f_s} \times \frac{d}{a} \times \frac{600}{600-F_y} \quad (63-3)$$

$$f_s = \frac{d}{a}(600 - F_y) - 600 \leq F_y \quad (64-3)$$

It should be noted that if compressive reinforcement is not required, this constraint is taken as 1 to avoid any disruption in the algorithm process.

- **Maximum Tensile Reinforcement:** The cross-sectional area of tensile reinforcement must be less than the maximum value specified by the code. This constraint is also dependent on the presence of compressive reinforcement.

### – Tensile Reinforcement:

$$\rho \leq \rho_{max} \rightarrow \rho - \rho_{max} \leq 0 \quad (65-3)$$

$$\rho_{max} = 0.75\rho_b \quad (66-3)$$

### Tensile and Compressive Reinforcement -

$$\rho \leq \overline{\rho}_{max} \rightarrow \rho - \overline{\rho}_{max} \leq 0 \quad (67-3)$$

$$\overline{\rho}_{max} = \rho_{max} + \rho' \frac{f_{sb}}{F_y} \quad (68-3)$$

**Minimum Effective Depth of Foundation:** The code defines a minimum effective depth of 15.24 cm (6 inches) for the foundation.

$$d \geq d_{min} \rightarrow d_{min} - d \leq 0 \quad (69-3)$$

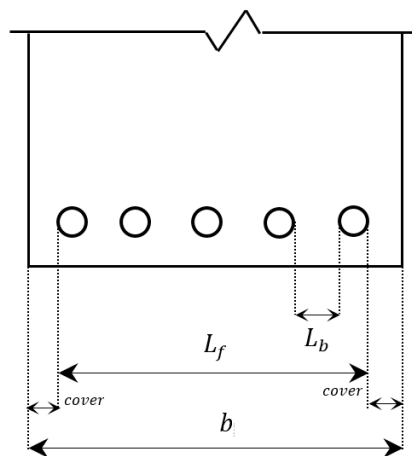
$$d_{min} = 15.24cm \quad (70-3)$$

**Stem Deflection Control:** To prevent deflection, the inclination of the stem with respect to the vertical must be greater than 0.02.

$$\tan \alpha \geq 0.02 \rightarrow 0.02 - \tan \alpha \leq 0 \quad (71-3)$$

$$\tan \alpha = \frac{B_s - t_t}{H_s} \quad (72-3)$$

As explained at the beginning of this section, the number of reinforcement bars is determined as a software output while complying with the minimum and maximum spacing requirements specified by the code. The spacing between the rebars, as shown in Figure 3-8, is calculated as follows.



**Figure 3-8:** Longitudinal Rebar Spacing in the Wall

$$L_f = b - 2cover \quad (73-3)$$

$$L_b = \frac{L_f - (n \times d_b)}{n-1} \quad (74-3)$$

Where  $nn$  is the number of rebars across the width, i.e., along the 1-meter length of the wall, which is calculated based on the cross-sectional area and the diameter of the bars.

**Minimum Rebar Spacing:** According to the code, the spacing between rebars must be greater than the larger of 2.5 cm or the bar diameter.

$$:L_b \geq \max(2.5cm, d_b) \rightarrow \max(2.5cm, d_b) - L_b \leq 0 \quad (75-3)$$

**Maximum Rebar Spacing:** This spacing must not exceed the lesser of 45.72 cm (18 inches) or three times the wall thickness.

**Stem**

$$L_b \leq \min(45.72cm, 3B_s) \rightarrow L_b - \min(45.72cm, 3B_s) \leq 0 \quad (76-3)$$

**Heel and Toe**

$$L_b \leq \min(45.72cm, 3D_b) \rightarrow L_b - \min(45.72cm, 3D_b) \leq 0 \quad (77-3)$$

It should be noted that, as previously mentioned, a value of  $-1$  is assigned for compressive reinforcement in cases where it is not present.

### 3-3-4 Penalty function

As stated at the beginning of the chapter, the responses obtained during the algorithm process are evaluated, and if the constraints are not satisfied, they will be penalized by the penalty function. The mentioned penalty function is defined as follows.

$$sum = sum + \max(0, g_i(x)) \quad (78-3)$$

$$f(x) = f(x) + RR \times sum \quad (79-3)$$

Where  $RR$  is a very large value that depends on the conditions of the problem and does not have a fixed number. For this reason, in this study  $10^{10}$  It has been assumed . At the beginning,  $sum$  value will be equal to zero. If none of the constraints  $g_i(x)$  become negative, the maximum value in equation (3-78) will no longer be zero. In this case, the value of the sum will also change, and the objective function in equation (3-79) will be increased by a very large amount. This penalty on the objective function causes incorrectly chosen points to be pushed away from the optimal

solution. (79-3) It will be added with a very large value. This penalty in the objective function causes the incorrectly selected points to be pushed away from the optimal solution.

## References

- [1] Saedi, A. Vadi, May 2015. Friction Dampers, Daryan Engineers Group, [www.daryan.com](http://www.daryan.com).
- [2] Hashemzadegan, M. and Abbasi, H., 2018. Investigation of the Effects of Friction Dampers with Brake Pads on the Behavior of Structures with Chevron Bracing. Islamic Azad University, Tehran South Branch. Second International Congress on Science and Engineering, Germany-Hamburg.
- [3] Khaleghian Farshid and Mohsen Tehranizadeh, 2007. Design of a New Type of Friction Damper with Brake Pads. Journal of Seismology and Earthquake Engineering (JSEE), Issue 4.
- [4] Karami Saeed and Abbas Haqollahi A., 2012. Presentation of a New Method for Distribution of Slip Load in Friction Dampers FBP, Sharif Civil Engineering Journal (Summer 2014), Vol. 2-30, No. 2, pp. 123-125.
- [5] Zahraei, M. and Sharifi, H. Presentation and Application of a New Type of Friction Damper with Metal Profile Parts and Brake Pads, Civil Engineering and Infrastructure Management, Faculty of Civil Engineering, University of Tehran and Tafresh University.
- [6] Saman Bagheri et al., 2015. Determination of Slip Load Characteristics of Friction Dampers on Different Floors of Building Frames Based on Target Ductility. Journal of Civil and Environmental Engineering, Vol. 45, No. 2..
- [7] Vatar, M.Q. and Abbasi, A., 2019. Performance Evaluation of Friction Dampers with Brake Pads Based on Laboratory Results. Tenth National Conference on Structures and Steel, Olympic International Conference Center.
- [8] Guide to Methods and Techniques for Seismic Retrofitting of Existing Buildings and Implementation Details, Publication No. 524.
- [9].Ana F. Santos , Aldina Santiago , Gianvittorio Rizzano , (2019) , Experimental response of friction dampers under different loading rates , International Journal of Impact Engineering , journal homepage : [www.elsevier.com/locate/ijimpeng](http://www.elsevier.com/locate/ijimpeng).
- [10].Ana Francisca Santos , Aldina Santiago , Massimo Latour , Gianvittorio Rizzano ,(2019), Analytical assessment of the friction dampers behaviour under different loading rates , Journal of Constructional Steel Research.
- [11].Sergio Pastor Ontiveros-Pérez , Letícia Fleck Fadel Miguel , Jorge Daniel Riera , (2019) , Reliability-based optimum design of passive friction dampers in buildings in seismic regions , journal Engineering Structures .