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Application of multi-objective optimization algorithms in Continuous Streamflow Modelling of a Mountain Subcatchment

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ABSTRACT

In recent decades, with significant advancements in data science and optimization algorithms, the use of novel methods to solve complex civil engineering problems has increasingly gained attention. One such method is the Giza Pyramid Construction Algorithm (GPC), which, following the construction process of the Giza pyramids, has been applied to optimize complex problems in various fields, including structural engineering. The Giza pyramids, architectural and engineering masterpieces from ancient times, demonstrate the high skill and knowledge of the engineers of that era in the optimal use of resources and mastery of structural dimensions. This strategy, from an algorithmic perspective, can assist contemporary optimization methods and structural analysis. By simulating the construction process of these pyramids, the mentioned algorithm enables civil engineers to provide intelligent and effective solutions in structural design, analysis, and optimization problems. This research investigates the applications of the Giza Pyramid Construction Algorithm in civil engineering optimization and will attempt to show how this algorithm can help improve efficiency, reduce costs, and increase safety in construction projects. Subsequently, the theoretical foundations, methodology, and related case studies of this algorithm will be examined to illustrate its significance and potentials in civil engineering.

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Introduction

For modeling complex systems like the SarouqChay subcatchment, a tool is necessary to provide a simple understanding of the system. The model can be considered as a theory, law, or structured idea, with the development of which, it is possible to understand complex systems. So far, various rainfall-runoff models have been developed to model this subcatchment. Generally, the choice of a rainfall-runoff model to estimate runoff values at the subcatchment outlet depends on the purpose of modeling and available data. Preparing the model (calibration), that is to determine the values of the parameters, is one of the major challenges. Calibration of rainfall-runoff models is usually carried out manually, automatically or the combination of both. Due to the time consuming of the manual calibration, the use of automatic calibration methods has been preferred. In this regard, the aim is matching the hydrograph calculated by the model with the observed hydrograph. That is, to simulate correctly the various characteristics of the hydrograph, including the high flows, low flows (both are called as extreme events) and the general shape of the hydrograph at subcatchment outlet (Sadeghi-Tabas, et al., 2017). In the meantime, the use of optimization algorithms as basic tools, to be both economic and manageable, are popular. Many of these algorithms are inspired by nature and operate on the basis of a variety of random search methods which are known as fractional algorithms. Although using these algorithms is not guaranteed to reach absolute optimal values, it is possible to be very confident in obtaining a near optimal solution. The studies have already been done in this regard are as follows.

Madsen (2000) used a Shuffled Complex algorithm for multi-objective optimization of the MIKE11 NAM rainfall-runoff model. The purpose of this study was to formulate and analyze the calibration strategy for a rainfall-runoff conceptual model. To calibrate this model, data from five years of rainfall, evapotranspiration potential, average temperature and daily runoff were used in a subcatchment in Denmark. In this research, a large number of parameters were introduced, all of which provided satisfactory results. The reason for this is the multipurpose calibration and Pareto space calibration, which evaluates the specified objectives.

Qaderi et al. (2006) conducted an automatic calibration for a rainfall-runoff model using SCE optimization algorithm. In this study, a global optimization developed algorithm had been used that had high potential for finding global optimum points. The study area was Gamasiab subcatchment from Karkheh River. In this research, three criterions were used like normalized root mean squares, model efficiency index and normal mean error. The results obtained for the calibration and validation periods indicated the efficiency and stability of the developed algorithm for automatic calibration of rainfall-runoff model parameters of NAM.

Bekele and Nickolw (2007) calibrated the SWAT (a physical and semi-distributed model) using the multi-objective optimization algorithm of the NSGA-II. For the first time, they calibrated the SWAT model by a multi-objective optimization algorithm. In this research, two target functions including RMSE and LOGE were used. Finally, it was reported that this optimization algorithm and selected target functions improved the performance of the model and obtained acceptable results from the Pareto optimal area.

Zhang et al. (2008) investigated and compared the five optimization algorithms like genetic optimization, particle swarm, differential evolution, randomized development of communities and artificial immune systems in calibrating a hydrologic model. By evaluating the target functions

over 2000 times, the genetic algorithm yielded better results and particle swarm algorithm showed better results for less value than that too.

Haung et al. (2012) studied three effective algorithms for multi-objective optimization algorithms called MOPSO, NSGA-II and MOSCEM-UA. These three algorithms were used on the Hanjiang River subcatchment by joining the HYMOD model. After running the algorithms, however, the MOPSO algorithm was more successful in finding the global optimum.

Khazaei et al. (2014) calibrated the ARNO Rainfall-Runoff conceptual Model using a simple genetic algorithm. With a sensitivity analysis technique, the intersection probability parameter was considered between 0.5 and 0.9.

In the present study, the subcatchment of SarouqChay located in the Urmiah Catchment in northwestern Iran was selected. The use of the ARNO rainfall-runoff model and the multi-objective optimization algorithms can be justifiable for estimating optimal parameters in this subcatchment. In this respect, this model is coupled with the three multi objective optimization algorithm (NSGA-II, SPEA-II and MOPSO) to calibrate its fifteen hydrological parameters.

2. Methodology

2.1. Data and Study Area

The SarouqChay Subcatchment (SCS) is located in upstream of Shahid Kazemi Dam (46°36′ to 47°23′ longitude and 36°12′ to 36°46′ latitude) with a drainage area of 2420 sqkm (figure 1). Due to completeness of its rain gauge data named TAKAB and its hydrometric station named Safakhaneh (See Figure 1). Daily rainfall, average temperature and discharge data from 1989 to 1999 have been used to simulate the streamflow in this study. The first 7 years is intended for the calibration of ARNO model and the 3 remained years were used for the validation of the model.

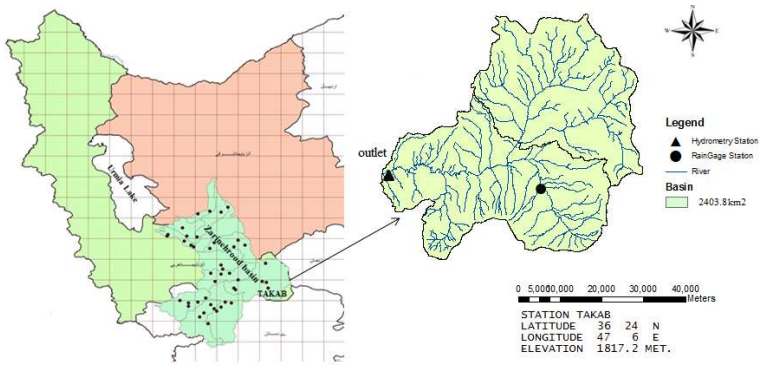

Figure 1. Location map of the SarouqChay Catchment (SCC)

2.2. The Arno Model

ARNO continuous rainfall-runoff model developed by Todini, (1988) and is used to simulate daily runoff in this study. This model is broadly used in water management, low flow, and real-time flood forecasting studies in different parts of the world. In ARNO, the basin is composed of an infinite number of elementary areas, each of which has a different soil moisture capacity. The spatial distribution of the soil moisture capacity is expressed in the form of a probability distribution function. Soil moisture content is fed by rainfall that infiltrates into the soil and is depleted by evapotranspiration, drainage and percolation. For each of the elementary areas with different soil moisture capacity and different soil moisture content, the

continuity of mass is simulated over time. Basin runoff is the integral of the runoff of elementary areas, transferred to the outlet of the basin via a routing module (Todini, 1996). In figure 2, ARNO model flowchart and its parameters are shown. The upper and the lower bounds that define the prior uncertainty ranges of the ARNO parameters are listed in Table 1. More details on the ARNO model are described by Todini (1996).

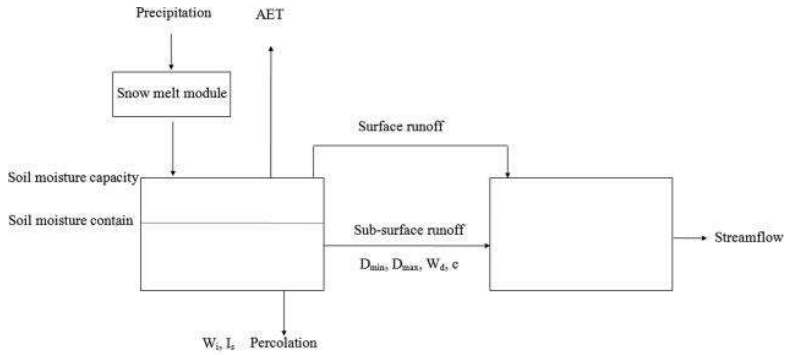

Figure 2. Diagram of the ARNO conceptual hydrologic model (Khazaei et al, 2012)

Table 1. The ARNO parameters used in this study

Parameter's Number	Parameter	Dimension and Symbol	Lower Bound	Upper Bound
1	Convectivity coefficient for the Hillslope	C1 [m/s]	1	3
2	Diffusivity coefficient for the Hillslope	D1 [m^2/s]	10	1000
3	Length of the path traveled by water to reach the channel	DX1 [km]	10	100
4	Convectivity coefficient for the channel	C2 [m/s]	1	3
5	Diffusivity coefficient for the channel	D2 [m^2/s]	100	10000
6	Length of the channel reach	DX2 [km]	1000	100000
7	Average volume of soil moisture storage	Wm [mm]	50	600
8	Moisture content threshold value in drainage calculation	SOL (or Wd) [mm]	0	300
9	Moisture content threshold value below which the percolation is negligible	SOL1 (or Wi) [mm]	0	100
10	A shape factor for the curve of soil moisture vs saturated areas	b	0.01	1
11	Maximum drainage that should be expected when the soil is completely saturated	Dmax [mm per time unit]	0	10
12	A drainage parameter	Dmin	0	10
13	Maximum percolation should be expected when the soil is completely saturated	PERC (or Is) [mm per time unit]	0	5
14	Exponent used to represent drainage when saturation is not reached	CESP (or c)	1.5	5
15	Initial volume of soil moisture storage	W0 [mm]	0	10

2.3. NSGA-II Algorithm

By far the most popular Evolutionary Multi Objective (EMO) these days is NSGA-II as Non-dominated Sorting Genetic Algorithm (Srinidhi et al., 2019). Here, introducing a dual measure used for comparing vectors modifies the selection. At first, a rank is assigned to each individual of the population. Individuals

currently in the Pareto set get rank 0 assigned. Then, starting with n = 0, iteratively all individuals of rank n are removed, and the rank (n + 1) is assigned to all individuals of the maximum set of the remaining set. This repeats until no more individuals without rank are available. For selection, individuals are first compared by their rank. But if the ranks are equal, a secondary measure is used, based on some other needs of the algorithm. In case of original NSGA-II, this is the so-called crowding distance. Its value is smaller the more the individual is residing in a part of the feasible space, where already other individuals of current population are located. Also here, the concept of ranking can be directly employed for fair dominance relations, as it is only referring to the maximum set of the relation. However, a corresponding complexity reduction introduced with NSGA-II for the rank computation is based on transitivity of the Pareto dominance relation, and this cannot be performed in case of fair dominance relations. The crowding distance is a plain computation from the vector components, without reference to a ranking relation.

2.4. MOPSO algorithm

Particle Swarm Optimization (PSO), has been modified into a multi-objective version in an increasing number of ways (so-called MOPSOs). In PSO, a swarm of particles is moving in a feasible space. Each particle maintains a locally best position of its own trajectory. and a global best position from the neighborhood of the particle (having a corresponding topology defined in advance). In each generation, positions of all particles are updated by a velocity term, which is the weighted sum of three components: the inertia term equal to former velocity; distance to local best, multiplied by a random number; and distance to global best, multiplied by a random number. For a multi-objective version, the ways of updating local and global best have to be modified, as simple numerical comparison of the fitness values is no longer possible. The update of the local best is typically based on the Pareto-dominance relation. Only if the particle after position update (and usually also a mutation) is dominating the former local best, it replaces the local best position. For global best selection, the algorithms usually maintain a set of mutually non-dominating positions, so-called leaders, and some means to select among the leaders and to update the leaders. In the simplest way, the leaders update is handled like an archive of an EMO (this concept is also employed here), and the leader selection is random. For extension to fair dominance case, similar arguments as for PAES apply: we can handle the set of leaders like the PAES archive, base the local best update on the fair dominance relation, and do not need adjust any other part of the MOPSO algorithm (Srinidhi et al., 2019).

2.5. SPEA-II Algorithm

In the Strength Pareto Evolutionary Algorithm (Zitzler et al., 2001), the basic concept for providing a modified selection is to assign a “Pareto strength,” or S-value to each member of the population. At first, for each individual i of current population, the number Ri of other individuals dominated by individual i is counted. In the second step, for each individual i the sum of all Rj values for individuals j dominating individual i is computed as Si. Then, the Si values are used like a fitness in single objective case for the other genetic operators. It can be immediately seen that this way of computing S values is not depending on the used relation.

2.5. Performance Criteria and Objective Functions

In this research, two objective functions were used to quantify the goodness of calibration performance: a LOGE (emphasizes low flow errors), and RMSE ((emphasizes peak flow errors) (eq and).

After running rainfall-runoff model with each algorithm, simulated flow obtained from the best set of parameters (selected by Kalai-Smorodinsky (KS) method) compared with observed streamflow. Statistical indicators including R-square (R2, Eq. (5)) and Nash- Sutcliffe (NSE, Eq. (7)) criteria were used to compare the performance of calibration.

$$LOGE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(LOG(\frac{\mathrm{Qo}_i}{\mathrm{Qs}_i}))^2} \quad (1)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Qo_i - Qs_i)^2} \quad \text{Eq} \quad (2)$$

$$R^2 = [\frac{\sum_{i=1}^{n}(Q_{obs,i} - \bar{Q}_{obs,i})(Q_{sim,i} - \bar{Q}_{sim,i})}{\sqrt{\sum_{i=1}^{n}(Q_{obs,i} - \bar{Q}_{obs,i})^2 \cdot \sum_{i=1}^{n}(Q_{sim,i} - \bar{Q}_{sim,i})^2}}]^2 \text{ Eq. (6)} \quad (3)$$

$$NSE = 1 - \frac{\sum_{i=1}^{n}(Qs_i - Qo_i)^2}{\sum_{i=1}^{n}(Qo_i - \overline{Qo})^2} \quad \text{Eq. (7)} \quad (4)$$

where Qo_i is the observed discharge, $\overline{Qo}$ is the mean value of observations, Qs_i is the simulated discharge, n is the number of observations, cc is the linear correlation coefficient between Qo and Qs, α is the ratio of standard deviation of Qs to standard deviation of Qo, and β is the ratio of the mean of Qs to mean of Qo. The smallest possible RMSE along with KGE and NSE close to 1, represent a better performance of likelihood function.
Also

3. Results and discussion

As it was mentioned, due to differences between the objectives of the question, selecting the corresponding point with a series of optimal parameters values from the Pareto optimal front between winner members is considered as an incompatible problem. The game theory can be used to select this point. In this study, the answer was selected using the Kalai-Smorodinsky solution method (KS) from obtained Pareto front. In Figure 4, the Pareto Front results from the optimization of the ARNO rainfall-runoff model using the NSGA-II, SPEA-II and MOPSO algorithms and the selected response is presented. Also, in Figure 5, the comparison of all three Pareto optimal fronts is shown.

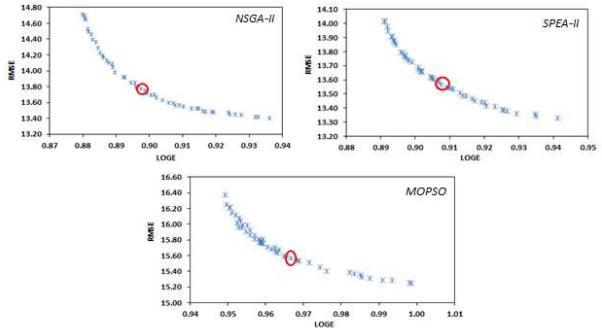

Figure 4. Optimal pareto fronts in the NSGA-II, SPEA-II and MOPSO methods

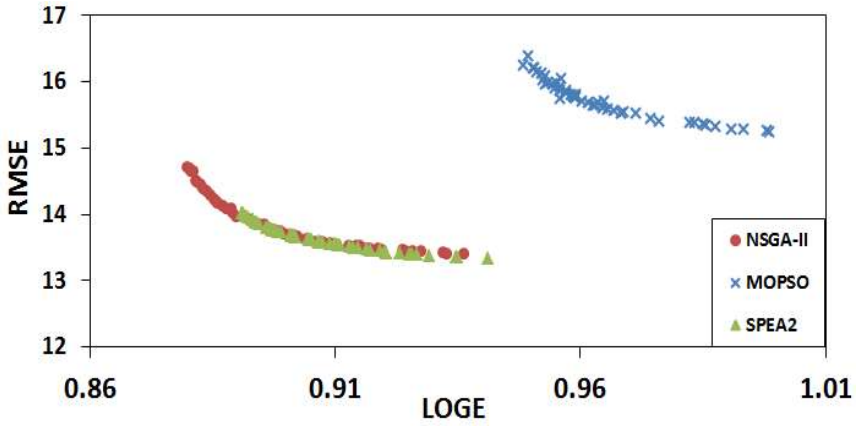

Fig. 5. Comparison of the Pareto optimal front in two algorithms

Using the best values of the parameters (selected using KS in each optimization algorithm), the computational hydrograph was calculated in calibration and validation periods. In the following, the optimal values of parameters in the three mentioned algorithms are shown in Table 1.

Table 1. Optimal Values of Parameters for the Selected Response from the Pareto Fronts

Number	Dimension and Symbol	Lower Bound	Upper Bound	NSGA-II	SPEA-II	MOPSO
1	C1 [m/s]	1	3			
2	D1 [m^2/s]	10	1000			
3	DX1 [km]	10	100			
4	C2 [m/s]	1	3			
5	D2 [m^2/s]	100	10000			
6	DX2 [km]	1000	100000			
7	Wm [mm]	50	600			
8	SOL (or Wd) [mm]	0	300			
9	SOL1 (or Wi) [mm]	0	100			
10	b	0.01	1			
11	Dmax [mm per time unit]	0	10			
12	Dmin	0	10			
13	PERC (or Is) [mm per time unit]	0	5			
14	CESP (or c)	1.5	5			
15	W0 [mm]	0	10			

For visual comparison, two obtained hydrographs from computational and observational runoff are presented for the obtained response from the Kalai-Smorodinsky method in the calibration and validation periods in Figures (6) to (8).

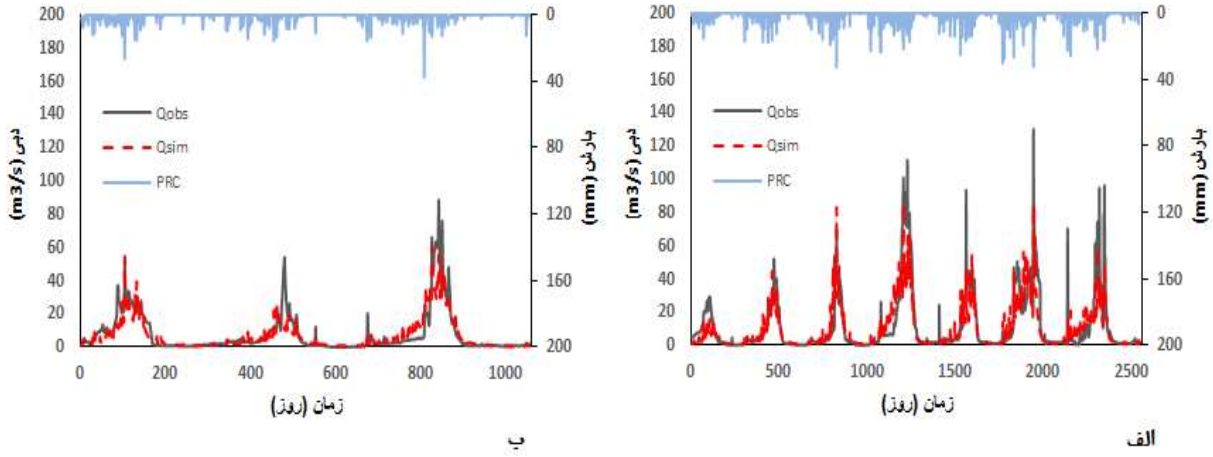

Figure 6. a- Observational and computational hydrograph in calibration period (1989-1995) b- Observational and computational hydrograph in validation period (1999-1996) using the NSGA-II algorithm

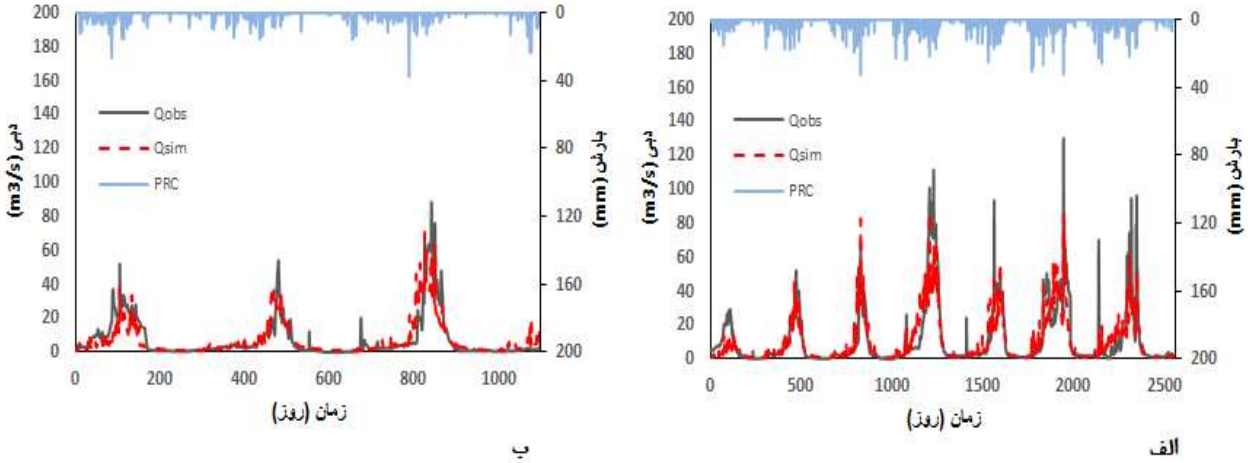

Figure7. a- Observational and Computational Hydrograph in calibration period (1989-1995) b- Observational and computational hydrograph in validation period (1996-1996) using the SPEA-II algorithm

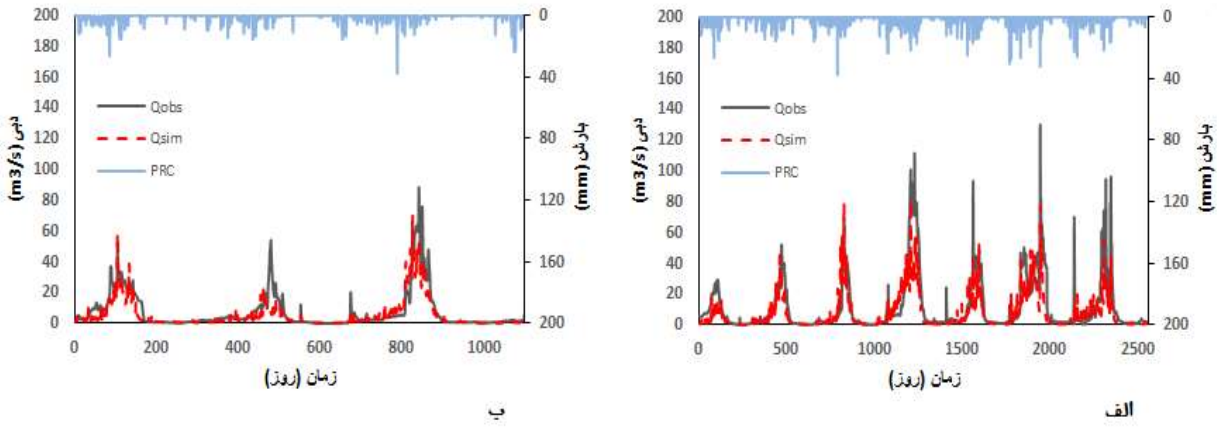

Figure 8. a- Observational and computational hydrograph in calibration period (1989-1995) b- Observational and computational hydrograph in validation period (1996-1996) using the MOPSO algorithm

Now in Table (2), the model is evaluated in the calibration and validation periods according to the performance critera.

Table 2. Evaluation Indexes in calibration and Validation Periods

	Calibration			Validation		
P-Criteria	NSGA-II	SPEA-II	MOPSO	NSGA-II	SPEA-II	MOPSO
R2	0.77	0.74	0.74	0.73	0.74	0.68
NSE	0.75	0.73	0.70	0.70	0.72	0.60

As you can see, the NSGA-II and SPEA-II algorithms have appropriate performance and they are close together (for both model evaluation index and for both calibration and validation periods). This issue can be verified in two parts: 1- These two algorithms present the same values for the optimal values of parameters. 2- In figure 5, which shows the Pareto fronts in both algorithms, and the values of the objective functions are close together.

4. Conclusions

This research was the first to optimize ARNO model parameters by multi-objective optimization algorithms. Regarding the existence of contradictory objective and the use of multi-objective optimization algorithms, a set of optimal answers was obtained instead of a unique optimal answer. The Kalai-Smorodinsky method was used to select a series of optimal parameters and the corresponding hydrographs were drawn. In the calibration period for the NSGA-II algorithm, the values of R^2 and the Nash-Sutcliff coefficient were 0.73 and 0.7, respectively. For the SPEA-II algorithm were 0.74 and 0.72, respectively. For the MOPSO method were 0.68 and 0.6, respectively. Also In the validation period for the NSGA-II algorithm, the values of R^2 and the Nash-Sutcliff coefficient were 0.77 and 0.75, respectively, for the SPEA-II algorithm were 0.74 and 0.73, respectively, and for MOPSO, these values were 0.74 and 0.70, respectively. The results showed that the presented algorithms had a good performance for simulating outflow runoff from the SaruqChay sub-catchment.

1. By investigation of the Pareto optimal front elongation, the NSGA-II algorithm is more successful than the other two algorithms. From the point of view of this index, the MOPSO algorithm had more acceptable performance than the SPEA-II algorithm. Therefore, according to the results of

this index, researchers and decision makers choose Optimal Values of Parameters in the NSGA-II algorithm compared to the other two algorithms.

2. By investigation of the number of dominant responses of an algorithm compared to the other, the performance of the NSGA-II algorithm is more appropriate than the other two algorithms. It should be noted that from the point of view of this index, other two algorithms overcame all of the obtained solutions from the MOPSO algorithm. Therefore, the use of Optimal Values of Parameters is recommended by two algorithms NSGA-II and SPEA-II.

3. 1. There are some calibration limits for the ARNO Rainfall-Runoff conceptual Model in the Saruq-Chay sub-catchment: a) If input data such as precipitation, temperature and potential evapotranspiration have errors, automatic calibration may physically offer unacceptable values. Therefore, after obtaining calibration and validation results, even after proper observational and computational hydrographs, the values of the parameters after the optimization are interpreted. b) The routing parameters in the SaruqChay subcatchment - an area of 2420 square kilometers –have a special complexity (In this model, six parameters are considered.). Since the corresponding sub-catchment is not very small to abandon the routing parameters easily and also is not large enough to have such a great effect on the performance of the model. It should be noted that the deletion of these parameters can result in the displacement of the peak times in the simulated hydrograph and thus results in weaker results than the model. It is suggested that the Sensitivity analysis of the model be calibrated with sensitive parameters in future research.

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