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Optimization of the Mix Design of High-Performance Concrete Containing Microsilica and Superplasticizer

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ABSTRACT

With the ever-increasing use of concrete in construction projects, the demand for concrete with improved properties is continuously growing. In this context, one of the concretes commonly used in specialized structures where high strength is required is High-Performance Concrete (HPC). Among the pozzolanic materials employed to enhance the strength and durability of concrete, microsilica (silica fume) has demonstrated excellent performance. The aim of this study is to optimize the mix design of concrete incorporating microsilica and a superplasticizer. To achieve this goal, an optimization model is first formulated for the mix design of this type of concrete. The development of the model is based on optimization principles combined with experimental results. An effective algorithm is then applied to solve the mix design optimization problem. The proposed method not only automates the mix design process for microsilica- and superplasticizer-based concrete but also minimizes the overall production cost of this type of high-performance concrete.

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1. Introduction

According to ASTM C618, a pozzolan is a siliceous or siliceous-aluminous material that, by itself, possesses little or no cementitious value, but, when finely divided, reacts with calcium hydroxide at ambient temperature in the presence of moisture to form compounds having cementitious properties similar to those produced by the hydration of Portland cement [1]. Therefore, a pozzolan is a natural or artificial material containing reactive silica. *Silica fume*, or microsilica, is a by-product of electric arc furnaces in ferroalloy and ferrosilicon industries and is a highly reactive pozzolanic material containing more than 85% amorphous silica. It is one of the most important and widely recognized industrial pozzolans.

With the ever-increasing use of concrete and advancements in optimization techniques in recent years, the optimization of concrete mix designs aimed at minimizing production costs has received considerable attention from researchers. Among the studies conducted on the optimization of high-performance concrete (HPC) mix designs, the work of Marsia et al. [2] is notable. They proposed a statistical mix design experiment to optimize the proportions of HPC. Shakhmenko and Brish conducted investigations on optimizing concrete materials using both analytical and numerical methods [3]. They addressed the appropriate aggregate gradation—a crucial aspect of concrete mix design—through two approaches: (a) using ideal grading curves, and (b) by theoretical and practical calculations of the added aggregate value.

Pekmezci and Akyuz investigated the optimal use of natural pozzolans to achieve maximum compressive strength in concrete [4]. Ozbay et al. employed the Taguchi method for mix design optimization [5], demonstrating the influence of different mix ratios on high-performance self-compacting concrete in both fresh and hardened states.

Additional studies on optimizing mix designs for various types of concrete used domestically include the following examples. Sadr-Karimi and Khaheshi conducted research on selecting the shape, type, and grading of aggregates for HPC [6]. Mostafinejad and Raeisi examined the effect of limestone powder on the compressive strength of microsilica-containing concrete and optimized the mix design using response surface methodology [7]. Habibi and Yousefi proposed a method for optimizing normal-strength concrete mix designs using the SLP algorithm, based on theoretical mix trials [8]. Habibi et al. further developed this method for concrete mix design based on optimization principles and laboratory results [9], showing that the optimal mix design corresponds to maximizing the largest coarse aggregate size and, consequently, the volume of coarse aggregate. Esmaeilnia Omran and Faridi constructed an optimized high-performance self-compacting concrete mix using the Taguchi approach [10]. Khokhar et al. conducted a study for optimization of mix design of concrete with high content of mineral additions to improve early age strength [11]. An optimization method based on Bolomey's law was used for the mix design of concrete to cope with the improper effect of replacement of cement by other materials at high percentages. Habibi and Ghomashi developed an optimum mix design method for self-compacting concrete based on experimental results [12]. The mix design was defined as an optimization problem by developing some relations for compressive strength and slump flow. They observed that the optimum self-compacting concrete mixes had minimum cost or cost-strength ratio as well as most suitable flowability and compressive strength. Khalid et al. used abundant database to intelligently select a mix-proportion of conventional OPC and fly ash (FA) based concretes [13]. First, data was sorted and bifurcated according to the grades of concrete, and a mean mix-design (MMD) approach was proposed for selecting constituents. Afterward, both data types were trained using an artificial intelligence (AI) tool, and strength was predicted for trained sets and testing sets of data. They observed that MMD, AI, and experimentation approaches have close correspondence for selected targeted strength.

A review of past studies indicates that the inclusion of microsilica has a positive effect on the mechanical properties, durability, and service life of concrete. Although extensive research has been conducted on traditional mix designs containing microsilica, studies on optimized mix designs for this type of concrete remain limited. Therefore, the objective of this research is to propose a method for optimizing high-performance concrete mix designs containing microsilica based on experimental results. To this end, an optimization model aimed at minimizing concrete production costs is first presented. Then, by establishing relationships between concrete strength and mix proportions derived from laboratory data, an optimized mix design method for microsilica-containing concrete is developed.

2. Optimized Mix Design Model for HPC

The optimization process is defined as determining the optimal values of design parameters (design variables) to minimize or maximize a certain quantity (objective function) subject to a set of specific design constraints. To develop an effective and suitable optimization model and algorithm for concrete mix design, its components—including design variables, objective function, and design constraints—must be clearly defined and formulated. In the following section, the details of the optimization model for the mix design of high-performance concrete are presented.

2.1 Design Variables

Parameters selected to describe a system are referred to as design variables, and variations in their values significantly affect the design outcome. The main constituents of the high-performance concrete considered in this study include water, cement, coarse aggregate, fine aggregate, and microsilica. Changes in the quantities of any of these components influence key properties of the mix, such as compressive strength. Under certain conditions, additional admixtures may be incorporated, and in this study, a superplasticizer is used as the admixture.

Therefore, the design variables in the optimization model for the concrete mix design are defined as follows: the weight of water (X_1), the weight of cement (X_2), the weight of coarse aggregate (X_3), the weight of fine aggregate (X_4), the weight of microsilica (X_5), and the weight of superplasticizer (X_6). These values are considered to define one cubic meter of fresh concrete.

2.2 Objective Function

For any system, there are numerous feasible designs, some of which yield better performance than others. To evaluate and compare different designs, a specific criterion is required. This criterion should be a scalar function of the design variables, calculated based on their values. Various objective functions can be defined depending on the design goal and the intended performance of the concrete structures. Examples of such objectives include resistance to sulfate and chloride attacks, target compressive strength, desired slump, and so on.

In this study, considering the high importance of minimizing construction costs, the production cost per unit volume of concrete is selected as the objective function to be minimized. Accordingly, the objective function for the mix design problem can be formulated as follows:

$$F(X) = C_1X_1+C_2X_2+C_3X_3+C_4X_4+C_5X_5+C_6X_6 \tag{1}$$

In the above equation, C_1, C_2, C_3, C_4, C_5, and C_6 represent the unit costs of water, cement, coarse aggregate, fine aggregate, microsilica, and superplasticizer, respectively. All the variables in Equation (1) and in the subsequent formulations are design variables. The cost coefficients used in Equation (1) may vary depending on the current economic conditions and can differ across countries and over time.

2.3 Design Constraints

To define an optimization model, the mix design is subject to a set of constraints, referred to as design constraints. A design solution is considered acceptable only if the design variables satisfy all the specified constraints. The optimization of a concrete mix design also involves secondary variables, such as the maximum size of aggregates, which are dependent on the primary design variables. For practical implementation and production of concrete based on the optimized mix design, the values of these variables should be specified in tables. Accordingly, the conditions for the design variables at each stage of the mix design must be simulated following ACI 211 [14]. The tables in this standard also include practical limitations and other bounds. Based on this, the design constraints can be defined as follows:

The compressive strength constraint is the first and most important restriction applied to the model, ensuring that the target characteristic strength is achieved for the optimized mix design. To impose this constraint, a relationship between the desired compressive strength and the design variables must be established, as described below. Thus, the compressive strength control constraint can be formulated as:

$$f_c \geq f_{ce} \tag{2}$$

where f_c is the compressive strength of the mix, and f_{ce} is the expected 28-day compressive strength of a cylindrical specimen.

Since the mix proportions are determined for one cubic meter of concrete, this limitation should be controlled through a design constraint expressed as:

$$A_1X_1+A_2X_2+A_3X_3+A_4X_4+A_5X_5+A_6X_6 \leq 1 \tag{3}$$

where the coefficients A_1-A_6 convert the weight of each component to its corresponding volume.

From a practical standpoint, the upper and lower bounds of each design variable are also considered as design constraints:

$$X_{il} \leq X_i \leq X_{iu} \tag{4}$$

where X_{il} and X_{iu} represent the maximum and minimum limits of the (i)-th design variable, respectively.

As observed, all mix design constraints, except for the first one (compressive strength), are explicit functions of the design variables and can be easily determined once the coefficients are known. One of the main challenges in achieving an optimal mix design, as noted in many previous studies, is the lack of a clear function for predicting the compressive strength of concrete. In this study, to address this issue, an explicit relationship is derived for the compressive strength constraint. In the following section, an effective method for formulating and establishing this constraint based on experimental results is presented.

2.3.1. Formulation of the Compressive Strength of HPC

As discussed in the previous section, a constraint that ensures the minimum 28-day compressive strength must be defined as a primary limitation in the optimization model. For this purpose, the following nonlinear relationship is proposed in the present study to estimate the characteristic compressive strength of the concrete mix:

$$f_c = a_0 + \sum_{i=1}^{6} a_i x_i + \sum_{i=1}^{6} a_{ii} x_i^2 \tag{5}$$

where a_0, a_i and a_{ii} are unknown coefficients determined through the processing of experimental data. The identification of appropriate coefficients for the proposed relationship—so that the estimation error of compressive strength is minimized—constitutes an unconstrained optimization problem. In this case, the design variables of the optimization model correspond to the unknown coefficients introduced in Equation (5).

The percentage error in the predicted compressive strength can be obtained from the following relationship:

$$Error = 100(f_{ci} - f_{cei})/f_{cei} \tag{6}$$

Where *Error* represents the value of the objective function, f_{ci} denotes the compressive strength predicted by the proposed model, and f_{cei} is the compressive strength obtained from experimental results.

In this study, 23 experimental data points were utilized based on previous research works [16–23]. By minimizing the objective function, the unknown coefficients used in Equation (5) were determined. The calculated coefficient values are presented in Table 1.

The error values obtained from Equation (6) for estimating the compressive strength of each mix design were calculated and are summarized in Table 2. The results indicate that the maximum error occurs for Mix Design No. 1, with a value of 10.12%. This finding demonstrates that the proposed relationship in Equation (5) provides a reliable and accurate prediction of the compressive strength for microsilica-containing concrete mixes.

Table 1. Values of the coefficients used in Equation (5)

Coefficient	Value	Coefficient	Value
a_0	434/203	a_{11}	0/0053
a_1	-1/8505	a_{22}	0/0009
a_2	0/5746	a_{33}	0/0006
a_3	-0/9585	a_{44}	-0/0021
a_4	3/1534	a_{55}	-0/0342
a_5	6/4375	a_{66}	0/9695
a_6	-37/4693		

Table 2. Summary of the calculated compressive strength estimation errors for each mix design

Mix design	Water $\frac{kg}{m^3}$	Cement $\frac{kg}{m^3}$	Gravel $\frac{kg}{m^3}$	Sand $\frac{kg}{m^3}$	Microsilica $\frac{kg}{m^3}$	Superplasticizer $\frac{kg}{m^3}$	Compressive strength (Experimental) Mpa	Compressive strength (Eq. 6) Mpa	Error (%)
1	19/41	32/535	91/49	84/39	36/17	1/8	66/91	60/13	1/120
2	19/41	30/457	91/49	84/39	54/25	1/8	63/03	64/11	1/72
3	19/41	361/7	91/49	84/39	0	1/8	47/03	45/64	2/95
4	16/74	513	825	759	27	6	71/67	69/66	2/79
5	16/74	496/8	822	856	43/2	6	72/05	71/48	/7870
6	16/74	480/6	820	853	59/4	6	77/94	73/99	/0595
7	160	450	656	959	50	1/9	75	73/71	/7191
8	160	500	656	984	0	1/3	62	56/96	8/12
9	160	400	656	935	100	2/4	79	75/92	3/9
10	240	600	960	640	0	8	68	64/88	4/58
11	27/27	360	67/58	34/59	40	4/8	30/8	30	2/51
12	34/56	450	53/31	27/73	50	6	42/11	41/4	1/66
13	175	500	1203	647	0	8/17	58	55/15	4/89
14	149	465	1168	615	30	3/1	80/3	76/91	4/22
15	149	450	1168	615	45	3/7	84/2	78/7	6/44
16	149	495	1168	615	0	1/9	67/4	68/9	2/23

17	172	430	1030	687	0	1/6	61/8	58/48	/3625
18	164	441	1115	653	28	2/9	73/9	72/71	1/60
19	175	470	1203	647	30	9/78	65	63/819	1/81
20	175	450	1203	647	50	11/71	67/5	65/2	3/39
21	140	325/5	1052	74/33	24/5	6	51	47/96	5/95
22	140	315	1052	821	35	35	59/5	56/9	4/25
23	140	315	1052	73/18	35	6	53	51/35	3/10

2.3. Optimization Algorithm

In this study, the mix design problem of HPC, defined in the previous section, is efficiently solved by using SQP method. In each iteration of this technique, by a quadratic approximation of Lagrangian function and a linear approximation of constraints, the optimization sub-problems are formulated and solved as a quadratic programming problem. The solution of this quadratic programming sub-problem involves determining a search direction and a suitable step-length. The step-length parameter is used to have a global convergence, i.e., when starting from an arbitrary design, the final solution be a design that satisfies the Karush-Kuhn-Tucker (KKT) optimality conditions. The main convergence criteria in SQP are based on numerically satisfying the KKT conditions. When the KKT conditions are satisfied with a desired tolerance, the algorithm will stop. More details about this technique have been presented by Arora [15].

3. Numerical Studies

3.1 Optimized Mix Design for Concrete with a Compressive Strength of 35 MPa

To demonstrate the effectiveness of the proposed optimization approach for designing concrete mixtures containing microsilica, an optimal mix design was developed for a target compressive strength of 35 MPa as a numerical example. In this case, the unit cost of water was taken as the reference (1 unit), while the relative costs of other constituents were assumed as follows: cement – 60 units/kg, coarse aggregate – 8 units/kg, fine aggregate – 7 units/kg, microsilica – 20 units/kg, and superplasticizer – 10 units/kg.

Based on these assumptions, the mix design problem was formulated as a standard mathematical optimization model, in which the objective function (total cost) was minimized subject to the design constraints defined earlier. Based on the experimental data, the minimum values of the design variables — namely, the weights of water, cement, coarse aggregate, fine aggregate, microsilica, and superplasticizer — were considered to be 140, 307.448, 531.3, 273.7, 0, and 1.3 kg, respectively. Similarly, the maximum values of these constituents were assumed to be 346.5, 600, 1203, 984, 100, and 35 kg, respectively.

After formulating the problem, the Sequential Quadratic Programming (SQP) algorithm was employed to obtain the optimal solution. The results of the optimized mix proportions are summarized in Table 3.

Figure 1 illustrates the convergence history of the objective function throughout the design iterations. As shown, the optimization process converged after only nine iterations, confirming that the proposed method achieves a high convergence rate and computational efficiency for optimizing high-performance concrete mixtures containing microsilica.

Figure 2 presents the constraint violation history during the optimization process. It can be observed that after the first iteration, the constraint violation decreased sharply and then stabilized, approaching zero at the final optimal design.

Table 3. Optimization Results for the Concrete Mix Design with a Compressive Strength of 35 MPa

Water $\frac{kg}{m^3}$	Cement $\frac{kg}{m^3}$	Gravel $\frac{kg}{m^3}$	Sand $\frac{kg}{m^3}$	Microsilica $\frac{kg}{m^3}$	Superplasticizer $\frac{kg}{m^3}$	Compressive strength Mpa	Cost $\frac{unit}{m^3}$
140	307.448	531.3	384.8938	29.9198	1.3	35	26142.94

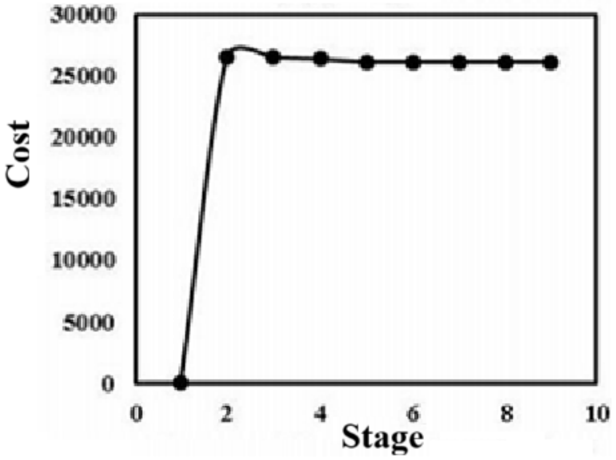

Figure 1. Convergence Trend of the Cost Function for the Concrete Mix Design with a Compressive Strength of 35 MPa

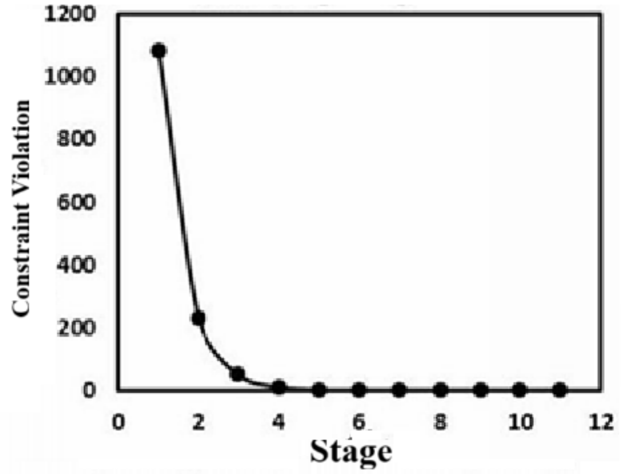

Figure 2. Constraint Violation History for the Concrete Mix Design with a Compressive Strength of 35 MPa

3.2. Optimized Mix Design for Concrete with a Compressive Strength of 65 MPa

In this numerical example, the mix design of microsilica-containing concrete with a target compressive strength of 65 MPa is analyzed. The objective of presenting this case is to compare the optimization results corresponding to two different target strengths. In this example, only the compressive strength constraint differs from the previous case, while all other design constraints, cost assumptions, and boundary conditions remain identical.

The optimized mix design corresponding to the 65 MPa strength level was determined using the proposed optimization approach, and the obtained results are summarized in Table 4. Figure 3 illustrates the evolution of the objective function during the optimization process. As shown, the proposed algorithm converged after 11 iterations, confirming its high efficiency and convergence rate in solving the optimal mix design problem for high-performance concrete. Figure 4 presents the constraint violation history, which, similar to the 35 MPa case, rapidly decreases after the initial few iterations and approaches zero by the 11th iteration, indicating complete convergence of the solution.

Table 4. Optimization Results for the Concrete Mix Design with a Compressive Strength of 65 MPa

Water $\frac{kg}{m^3}$	Cement $\frac{kg}{m^3}$	Gravel $\frac{kg}{m^3}$	Sand $\frac{kg}{m^3}$	Microsilica $\frac{kg}{m^3}$	Superplasticizer $\frac{kg}{m^3}$	Compressive strength Mpa	Cost $\frac{unit}{m^3}$
140	357.448	531.3	553.9958	59.5868	1.3	65	27919.99

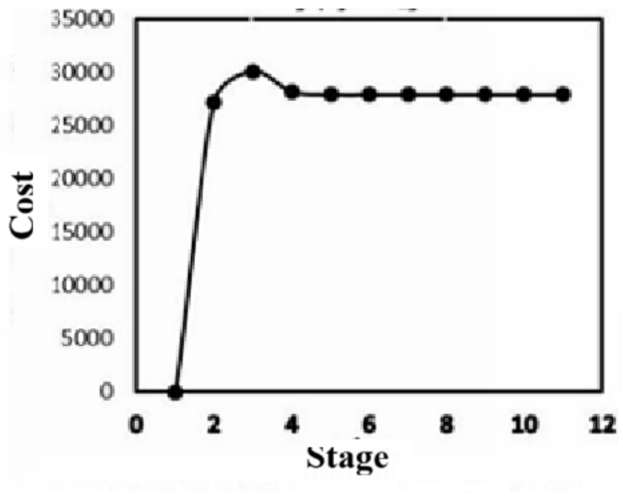

Figure 3. Convergence Trend of the Cost Function for the Concrete Mix Design with a Compressive Strength of 65 MPa

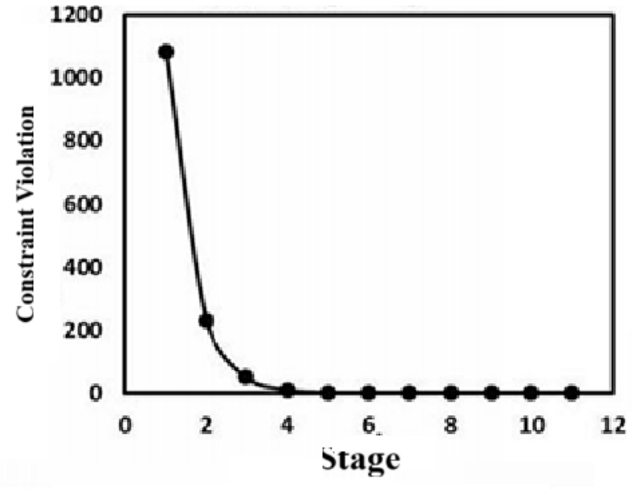

Figure 4. Constraint Violation History for the Concrete Mix Design with a Compressive Strength of 65 MPa

4. Validation of the Proposed Method

In this section, the results of the optimization process for high-performance concrete mix designs with target compressive strengths of 35 and 65 MPa, as presented in the previous section, are validated and evaluated. For this purpose, the optimized mix design results are compared with the experimental mix designs corresponding to the same strength levels listed in Table 2, in order to assess the improvement achieved by the proposed approach. This comparison is summarized in Table 5.

It should be noted that, for the 35 MPa concrete mix, since there is no experimental mix design with exactly the same compressive strength in Table 2, Mix No. 11, with a compressive strength of 30.8 MPa, was selected as the closest available case. When comparing the minimum unit cost of producing one cubic meter of concrete with the target strength of 35 MPa using the proposed optimization approach against that of Mix No. 11 (obtained using the classical design method), it is observed that, despite the higher compressive strength of the optimized mix, the total cost is approximately 14% lower.
This clearly demonstrates the effectiveness of the proposed method in achieving significant cost reduction compared to traditional mix design procedures.

For the second numerical case, corresponding to the 65 MPa concrete mix, Mix No. 19 from Table 2 — which exactly matches the target compressive strength — was used for comparison. As shown in Table 5, the optimized mix design obtained from the proposed method results in a 42% reduction in cost compared to the classical mix design (Mix No. 19). This substantial decrease in cost highlights the remarkable economic efficiency of the proposed optimization approach in practical applications.

Table 5. Comparison Between the Optimized and Classical Concrete Mix Designs

Mix design	Water $\frac{kg}{m^3}$	Cement $\frac{kg}{m^3}$	Gravel $\frac{kg}{m^3}$	Sand $\frac{kg}{m^3}$	Microsilica $\frac{kg}{m^3}$	Superplasticizer $\frac{kg}{m^3}$	Compressive strength Mpa	Cost (Units/m^3)
Classic (30.8 MPa)	277/2	360	678/5	349/5	40	4/8	30/8	30599/7
Optimum (35 MPa)	140	307/448	531/3	384/8938	29/9198	1/3	35	26142/94
Classic (65 MPa)	175	470	1203	647	30	9/78	65	43225/8
Optimum (65 MPa)	140	307/448	531/3	553/99	59/58	1/3	65	27920

5. Conclusion

In this study, a method was developed for the optimal mix design of high-performance concrete (HPC) incorporating microsilica and a superplasticizer. To achieve this goal, the concrete mix design problem was formulated as an optimization model, with the objective of minimizing the total production cost of concrete while satisfying all regulatory and practical constraints. Six design variables were considered, including the weights of water, cement, coarse aggregate, fine aggregate, microsilica, and superplasticizer. The imposed constraints included the target compressive strength, volumetric and weight limitations, and practical construction requirements, all of which were integrated into the optimization model. To establish the compressive strength relationship, experimental data from microsilica-based concrete samples were utilized. The Sequential Quadratic Programming (SQP) algorithm was employed to solve the formulated optimization problem.

The results demonstrated that the proposed method is capable of minimizing the production cost of microsilica-containing high-performance concrete while satisfying all code-based and practical limitations. The numerical studies indicated that the employed algorithm converges efficiently within a limited number of design iterations. Furthermore, the numerical findings revealed that the optimized mix design can lead to a significant reduction in cost compared to the traditional (classical) mix design approach.

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