Concepts and Functions in The Building Engineering

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Regression Analysis of NSM FRP and FRP Wrapping Effects in Strengthened RC Columns

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ARTICLE INFO

Article history:
Received: 15 December 2024
Revised: 24 January 2024
Accepted: 05 March 2024

Keywords:

RC Column, NSM FRP, FRP jacket, Axial Strengthening.

ABSTRACT

In this study, a simple regression-based approach is developed to accurately predict the axial load capacity of RC (Reinforced Concrete) columns strengthened using NSM (Near Surface Mounted) and hybrid FRP (Fiber Reinforced Polymer) methods. The model is trained and validated using a dataset of 112 experimentally tested specimens collected from the literature, including 22 data points from the authors' previous study and 90 additional cases. Key input parameters include column dimensions (width and height), concrete compressive strength (mean 35.1 MPa), longitudinal steel ratio (mean 1.43%), transverse steel ratio, number and properties of NSM FRP and FRP jackets, and load eccentricity ratio (mean 0.18). The dataset was split into 80% training and 20% test sets. Three regression models were developed: (i) full linear regression, (ii) regression with statistically significant features, and (iii) stepwise regression. The stepwise regression model showed the best performance with a Mean Absolute Percentage Error (MAPE) of 10.85% and Root Mean Square Error (RMSE) of 201.91 kN on the test set. This approach eliminates complex trial-and-error procedures and offers a transparent, practical tool for engineers and researchers involved in strengthening RC columns.

E-ISSN: 000-000

How to cite this article:
M.A. Shayanfar, (2024). Regression Analysis of NSM FRP and FRP Wrapping Effects in Strengthened RC Columns, 1(3), 1-١١. https://doi.org/

Introduction

Reinforced concrete (RC) structures are widely adopted in civil engineering due to their durability and cost-effectiveness. However, over time, factors such as changes in usage, design or construction flaws, updates in seismic or structural codes, and material degradation often necessitate retrofitting instead of full demolition and reconstruction. Opting for retrofit strategies can significantly extend a structure's service life while contributing to environmental sustainability by reducing demolition waste and conserving resources. Among the available strengthening techniques, fiber-reinforced polymers (FRP) have gained popularity due to their high strength-to-weight ratio, corrosion resistance, and ease of application [1]. Particularly for RC columns—which are the primary load-bearing elements—ensuring sufficient axial and flexural capacity is vital. While FRP jacketing improves compressive behavior through confinement [1], more complex loading conditions such as combined axial force and bending moments may benefit from the Near-Surface Mounted (NSM) technique or hybrid strengthening methods [14, 6-2].

The NSM technique involves embedding FRP bars into grooves cut in the concrete cover and bonding them with adhesives like epoxy or cement-based grout, primarily to enhance flexural and shear strength. Although ACI 440-17 [1] does not consider the compressive contribution of FRP in longitudinal applications, research has shown that it can influence axial capacity under eccentric loads [2–6]. Moreover, the bond between FRP and concrete is a critical factor in ensuring effective stress transfer and maximizing FRP efficiency, and is affected by parameters such as bar surface characteristics, groove dimensions, and concrete compressive strength [4, 17, 18]. Conventional analytical models based on strain compatibility and force equilibrium often fall short due to these nonlinear complexities and require iterative approaches [5, 6, 20]. In light of these challenges, this study develops a simple regression-based model to predict the axial capacity of RC columns strengthened with NSM FRP, aiming to offer a practical yet effective alternative to existing analytical approaches.

In this study, a simple regression-based approach is developed to accurately predict the axial load capacity of RC columns strengthened using the NSM and hybrid methods. Unlike recent works that often rely on synthetic datasets or complex machine learning models, the proposed model is trained and validated using a dataset composed entirely of experimentally tested specimens gathered from the literature. A total of 112 data points were used, including 22 from the authors' previous experimental study [4] and 90 additional cases collected from published sources. By focusing on regression analysis, this study eliminates the need for complicated trial-and-error procedures and aims to provide a transparent and practical tool for engineers and researchers. The next sections describe the dataset and the methodology employed for developing the model.

It is worth noting that the same experimental database employed in this study has also been utilized by the authors in a recently published ISI journal article [23]. In that work, several machine learning (ML) models—including Random Forest, Gradient Boosting, and Support Vector Regression—were developed and further optimized using metaheuristic algorithms to improve their hyperparameters. The results demonstrated the superior predictive accuracy of ML approaches in comparison with conventional regression models. To enrich the current conference paper and in line with the reviewers' valuable suggestions, the regression results presented herein are further compared with the ML outcomes reported in the aforementioned study. This comparison provides a clearer understanding of the advantages and limitations of regression-based models for practical engineering applications. It should also be emphasized that regression models, despite being less accurate than advanced ML approaches, retain the practical advantage of offering explicit closed-form equations, which makes them highly useful for engineers in real-world design and assessment tasks.

EXPERIMENTAL DATASET

The initial influential parameters recorded in this study include the width (b) and height (h) of columns cross section, the 28-day compressive strength of concrete (f_c'), the longitudinal steel ratio (ρ_g) and its yield stress (f_y), transverse steel area to their center to center distance ratio ($A_{v/s}$), column height (L), the number of NSM FRP (n_{NSM}), its modulus of elasticity (E_{NSM}) and its ratio (ρ_{NSM}), the number of layers (n_J), thickness (t_J) and modulus of elasticity (E_J) of FRP jacket, load eccentricity ratio (e/h), and maximum experimental compressive capacity (P_{exp}). The statistics of 112 collected dataset from open literature [2-13-22-23] is presented in Table 1, and their histogram diagram is represented in Fig.1.

Table 1. The statistics of the collected dataset

Parameter	Mean	Std	Min	Max
b (mm)	181.75	38.52	133.00	230.00
h (mm)	202.12	63.83	133.00	450.00
f_c' (MPa)	35.10	8.66	22.30	55.20
ρ_g (%)	1.43	0.35	1.00	2.56
A_v/s (mm)	1.25	0.83	0.28	3.77
f_y (MPa)	528.44	61.17	340.00	612.00
L (mm)	973.39	888.45	450.00	4100.00
n_{NSM}	3.43	3.39	0.00	12.00
E_{NSM} (GPa)	99.99	75.84	0.00	238.00
ρ_{NSM} (%)	0.48	0.51	0.00	2.00
n_J	0.97	1.13	0.00	4.00
t_J (mm)	0.09	0.07	0.00	0.20
E_J (GPa)	112.60	89.39	0.00	240.00
e/h	0.18	0.23	0.00	0.75
P_{exp} (kN)	1244.92	787.40	182.00	2925.00

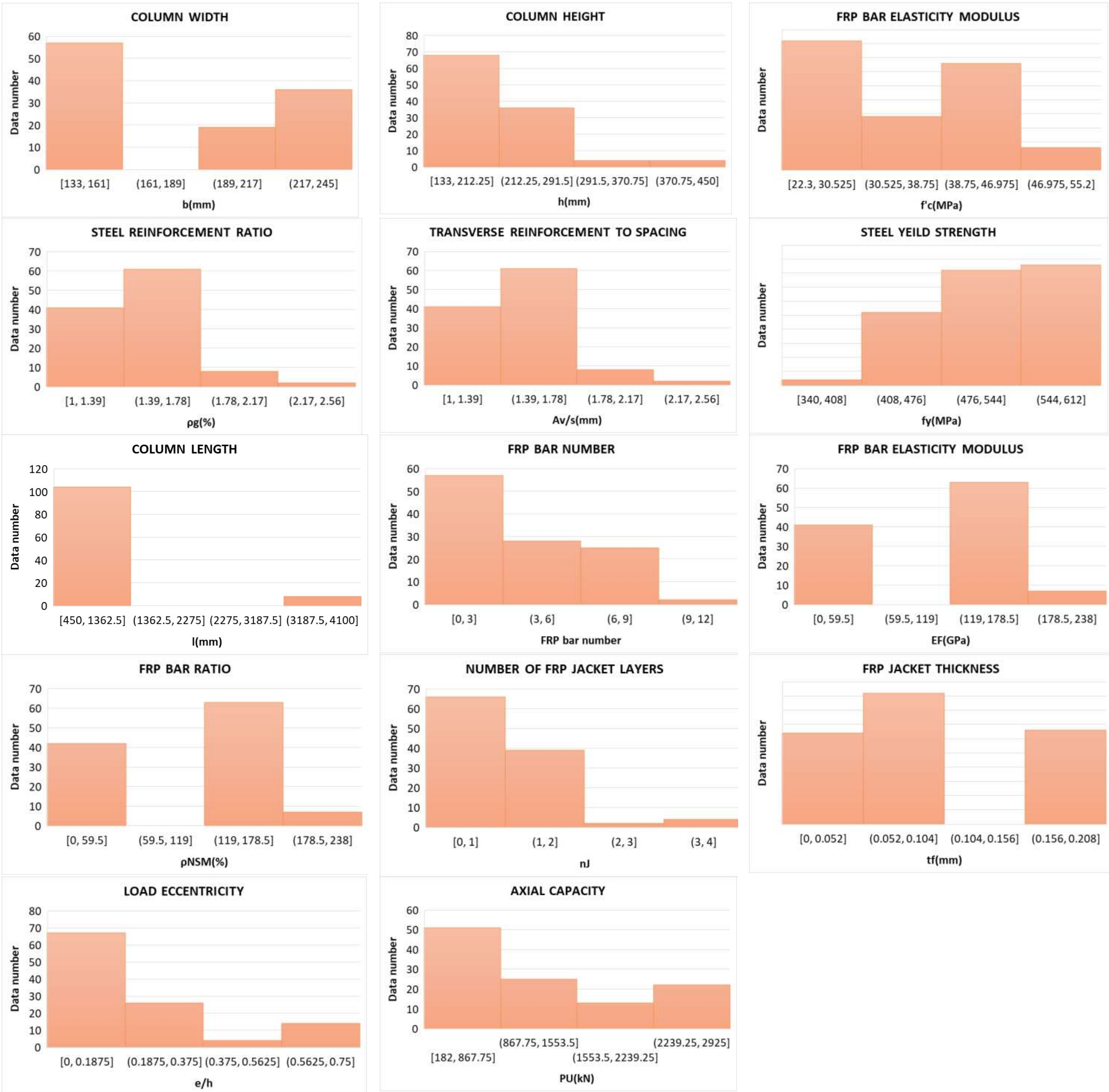

Figure 1- The Histogram Diagram of The Collected Dataset

METHODOLOGY

This section outlines the step-by-step procedure adopted in the study for developing and evaluating regression-based predictive models. The steps include data preparation, feature selection based on statistical significance, model development, and performance evaluation using standard error metrics.

3.1- DATA PREPARATION AND PARTITIONING

Let the dataset be represented as a matrix $D = [X|y]$, where:
$X \in R^{n\times p}$: matrix of p predictor variables (features)
$y \in R^{n\times 1}$: vector of response variables
n: number of observations
The dataset was randomly split into two mutually exclusive subsets:
Training set: 80% of data used for model training
Test set: 20% of data used for out-of-sample validation
This was implemented using the Hold-Out cross-validation strategy with a fixed random seed to ensure reproducibility:

$$D = D_{\text{train}} \cup D_{\text{test}}$$
$$D_{\text{train}} \cap D_{\text{test}} = \emptyset$$

3.2- Full Linear Regression Model

A full multiple linear regression model was fitted on the training data using all predictors:

$$y_i^{\wedge} = \beta_0 + \sum_{j=1}^{p} x_j \beta_{ij} + \varepsilon_i \tag{1}$$

where:
$y_i^{\wedge}$ is the predicted response, β_0 is the intercept term, β_{ij} are the regression coefficients, $\varepsilon_i \sim N(0, \sigma)$ are the residual errors.
The model was fitted using Ordinary Least Squares (OLS), minimizing the residual sum of squares (RSS):

$$RSS = \sum_{i=1}^{nTrain} (y_i - y_i^{\wedge})^2 \tag{2}$$

3.3- FEATURE SELECTION BASED ON STATISTICAL SIGNIFICANCE

To reduce model complexity and potentially improve generalization, statistical significance of predictors was evaluated based on their p-values in the full model. For each coefficient β_j, the null hypothesis $H_0 = \beta_{0j}$ was tested. Predictors with $p_{value}<0.05$ were considered statistically significant:

$$\left\{ j \in \{1,2,\dots,p\} \middle| p_{value}(\beta_j) < 0.05 \right\} = S \tag{3}$$

3.4- LINEAR REGRESSION WITH SELECTED FEATURES

A new linear regression model was trained using only the statistically significant features:

$$y_i^{\wedge} = \beta_0 + \sum_{j\in S}^{p} x_j \beta_{ij} + \varepsilon_i \tag{4}$$

This reduced model is expected to retain the most relevant information for predicting the response while reducing overfitting.

3.5- STEPWISE LINEAR REGRESSION

Stepwise regression was applied using the selected features as the initial candidate pool. This method iteratively adds or removes predictors based on Akaike Information Criterion (AIC):
Forward selection: adds variables that improve the model fit.
Backward elimination: removes variables that do not significantly contribute to the model.
The algorithm selects the model
where:

$$L^{\wedge} ln2 - 2k = AIC \tag{5}$$

k is the number of estimated parameters $L^{\wedge}$ is the maximized value of the likelihood function.

3.6- MODEL EVALUATION METRICS

Both the linear and stepwise models were evaluated using the following performance metrics:
Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - y_i^{\wedge})^2} \tag{6}$$

Mean Absolute Percentage Error (MAPE):

$$MAPE = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{y_i - y_i^{\wedge}}{y_i}\right| \tag{7}$$

Mean Error Ratio (MEAN):

$$MEAN = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i^{\wedge}}{y_i} \tag{8}$$

These metrics were computed for Training data, Test data, and All data (combined) for regression model with the whole predictors, and regression and stepwise regression model.

RESULTS AND DISCUSSION

4.1- CLOSED-FORM REGRESSION EQUATIONS

One of the strengths of linear regression models compared with machine learning methods is that they provide **explicit closed-form equations**, which allow engineers to directly estimate the target response without requiring trained models or computational environments.
Using the regression coefficients obtained from the statistical analysis in MATLAB, the following closed-form predictive equations were developed for the axial capacity of strengthened columns.

(1) Full Linear Regression Model (All Features)

$$\hat{y} = -1286.196 + 6.619x_1 + 4.363x_2 + 14.492x_3 - 339.597x_4 + 395.560x_5 + 1.020x_6 - 0.496x_7 + 44.485x_8 - 0.549x_9 - 121.505x_{10} + 75.433x_{11} + 818.389x_{12} + 0.0319x_{13} - 1746.491x_{14} \quad (9)$$

(2) Reduced Linear Regression Model (Selected Features)

$$\hat{y} = -1282.203 + 8.088x_1 + 4.960x_2 + 18.938x_3 - 284.390x_4 + 236.807x_5 - 0.351x_6 + 18.720x_7 + 97.814x_8 - 1737.835x_9 \quad (10)$$

(3) Final Stepwise Regression Model

This model includes interaction terms as selected automatically via the stepwise procedure:

$$\hat{y} = -2811.513 + 7.704x_1 + 18.272x_2 - 19.955x_3 + 0.456x_6 + 21.429x_7 + 6.110x_8 + 2428.692x_9 - 14.363(x_1x_9) - 0.0151(x_2x_6) + 0.704(x_2x_8) + 0.06875(x_3x_6) - 29.406(x_3x_9) - 197.390(x_8x_9) \quad (11)$$

This stepwise model provided the highest predictive accuracy among all regression models, benefiting from the inclusion of statistically significant interaction terms.

4.2-Models Performance

In this study, three different models were developed to predict the response variable: Linear Regression using all 14 input features (Full Linear Regression), Linear Regression using selected features (based on $p_{value} < 0.05$), and Stepwise Regression using the selected features. The error metrics for each of the developed models over the training, test, and combined datasets are summarized in Table 2. These metrics include RMSE, MAPE, and Mean Ratio, providing a comprehensive comparison of the models' predictive performance.

Table 2. The error metrics for 3 developed models

Model	Dataset	RMSE	MAPE	Mean Ratio
Linear Regression (All Features)	Train	165.40	16.39%	0.9932
	Test	208.10	17.54%	1.1032
	All	174.61	16.62%	1.0148
Linear Regression (Selected)	Train	175.76	16.36%	0.9921
	Test	219.42	15.97%	1.0980
	All	185.15	16.29%	1.0129
Stepwise Regression (Selected)	Train	121.26	10.08%	1.0014
	Test	201.91	10.85%	1.0629
	All	140.80	10.23%	1.0135

Initially, 9 statistically significant features were identified based on the p-value criterion, which are: (b) and height (h) of columns cross section, the 28-day compressive strength of concrete (f_c'), the longitudinal steel ratio (ρ_g), transverse steel area to their center to center distance ratio ($A_{v/s}$), column height (L), the number of NSM FRP (n_{NSM}), the number of layers (n_J), thickness (t_J) and modulus of elasticity (E_J) of FRP jacket, load eccentricity ratio (e/h). These features were then used as inputs for the second and third models to reduce model complexity and potentially improve performance.
Performance evaluation metrics indicate that the Stepwise Regression model outperformed the other two models. Specifically, on the test dataset, it achieved a MAPE of 10.85% and an RMSE of 201.91. The mean prediction ratio was 1.0629, which is very close to the ideal value of 1.
To visually compare the performance of the models, the prediction results on the entire dataset are shown in the following Fig. 2.

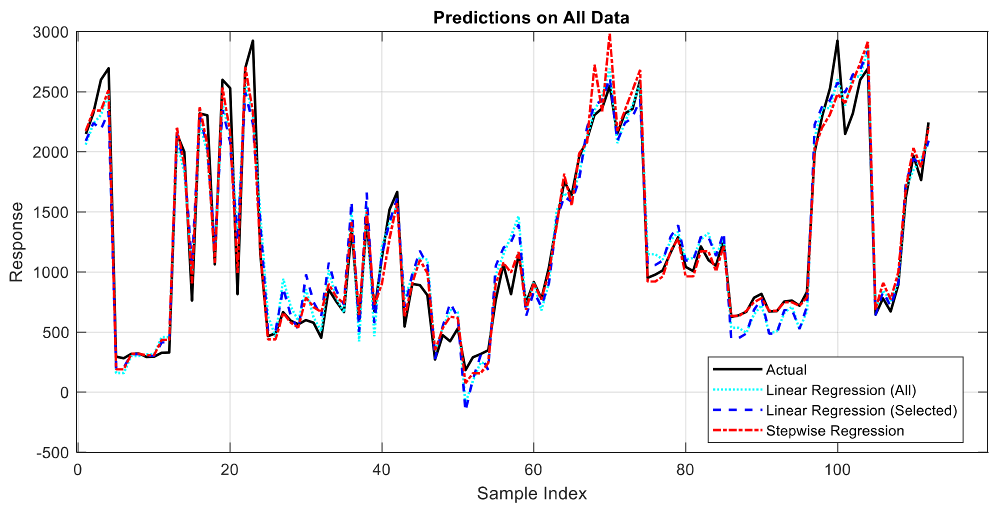

Figure 2- Prediction Performance of Regression Models on Entire Dataset

In this plot, the actual values (black line) are compared with the predicted outputs from each model. As observed, the Stepwise Regression model (red dashed line) closely follows the trend of the actual data and has the smallest deviations in most regions. In contrast, the linear regression model using all features (light blue dotted line) shows less accurate predictions in certain areas.
Overall, the results demonstrate that feature selection based on statistical significance, combined with stepwise modeling, can significantly enhance prediction accuracy. Therefore, the stepwise regression model with selected features is recommended as an efficient and reliable approach for similar predictive tasks.

4.3- Comparison with Machine Learning Models

To provide a broader perspective on the predictive capability of conventional regression models, the results obtained in this study were compared with the performance of machine learning (ML) models previously developed by the authors using the same experimental database [23]. In that ISI journal publication, multiple ML algorithms—including Random Forest, Gradient Boosting, Support Vector Regression, and several metaheuristic-optimized variants—were trained to predict the axial capacity of strengthened RC columns. The use of metaheuristic optimization significantly improved the hyperparameters of the ML models, resulting in substantially higher predictive accuracy compared with traditional regression approaches.

The comparison reveals that although the regression models developed in the present conference paper provide acceptable accuracy, their overall performance remains inferior to the ML models reported in the aforementioned study. This finding is consistent with the general expectation that ML techniques, due to their non-linear learning capabilities, can more effectively capture complex interactions within experimental data. Nevertheless, regression models retain their unique advantage of offering explicit closed-form equations, making them highly practical for engineering design, rapid assessment, and code-based applications where interpretability and transparency are essential.

CONCLUSIONS

- This study presented a regression-based approach to predict the axial capacity of RC columns strengthened with Near-Surface Mounted (NSM) and hybrid FRP techniques. The need for efficient and transparent predictive tools has grown as structural retrofitting becomes more common, particularly in aging infrastructure.

- A comprehensive dataset of 112 experimental cases was compiled from existing literature and previous experimental studies. Using this dataset, three types of regression models were developed and compared: (1) full linear regression, (2) reduced linear regression including only statistically significant variables, and (3) stepwise regression based on statistical criteria.

- Among the developed models, the stepwise regression demonstrated the best trade-off between simplicity and predictive accuracy. It eliminated redundant variables while maintaining performance, thus offering a practical and interpretable solution for engineering use.

- The findings of this study show that properly developed regression models can serve as reliable alternatives to more complex and less transparent machine-learning-based methods. Future research can focus on expanding the database, incorporating additional variables (such as confinement effects and failure modes), or applying hybrid regression techniques to enhance predictive capability and generalizability.

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