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Application of Utility Function for Optimization and Ranking of Concrete Mix Designs

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ABSTRACT

Calcium aluminate cement is considered a special cement due to its unique structure and the presence of at least 40% aluminum oxide. It is highly durable under severe environmental conditions. However, challenges such as high cost and more importantly, sudden reduction in strength have limited its use as a standalone cement without structural modifications through alternative materials. The wide variety of these materials and their diverse effects on the various properties of this cement increase the number of variables, often causing confusion among engineers when selecting materials in different proportions. In this study, 20 mix designs were prepared with replacement percentages of 40%, 25%, 15%, 5%, and 60% of alternative materials including zeolite, slag, pumice, and limestone powder, and compared with a control mix. Subsequently, using the utility function, a concept from mathematics, the mixes were ranked from technical and economic perspectives.

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Introduction

Concrete containing calcium aluminate cement has been used in civil engineering projects for over 100 years. This cement initially gained attention due to its high early strength in the construction of weapon storage facilities during World War I in France [1]. Today, it has various applications, including as a heat-resistant material in refractory industries [2-4] and as a material resistant to acidic and corrosive environments such as industrial floors and sewage pipes [5-6]. Another important advantage of this cement is its ability to gain rapid strength even at low temperatures (around zero degrees Celsius), which is why calcium aluminate cement is recognized in North America as a fast-setting repair material [7-8]. Despite its strengths, the widespread use of this cement is limited for two main reasons: its cost, which is about ten times higher than Portland cement, and a sudden reduction in strength over time.

A solution proposed by previous researchers to address these two weaknesses is the use of cement replacement materials [9]. In this study, as described below, 21 mix designs were prepared, various tests were conducted on them, and the cost of each mix design was calculated based on current market prices in Iran.

2. Materials and Methods:

Calcium aluminate cement is highly capable of durability under severe environmental conditions. However, as mentioned earlier, issues such as the high cost of the product and more importantly, the sudden reduction in strength, have limited the use of this cement alone without certain modifications in its structure by using replacement materials. One of the most important concerns in the production of any product or material today is cost reduction during the manufacturing process. However, ignoring technical parameters and focusing solely on cost reduction cannot lead to the production of a suitable and widely used product. Therefore, first and foremost, any product must be technically acceptable, which is a higher priority, and then cost considerations should be addressed.

The wide variety of materials and their diverse effects on various properties of this cement increase the number of variables and sometimes cause confusion among engineers when selecting materials at different proportions. In this study, after preparing 20 mix designs with replacement percentages of 40%, 25%, 15%, 5%, and 60% of alternative materials including zeolite, slag, pumice, and limestone powder, these were compared with a control sample. Then, using the utility function, the mixes were ranked from technical and economic perspectives. In this research, the code for each mix design indicates the type of material used and the replacement percentage of cement. For example, the code **C** refers to the control sample (without replacement materials), **Z25** indicates a mix with 25% zeolite replacement, **P** stands for pumice, and **L** and **S** represent limestone powder and slag, respectively. In this study, mechanical properties, durability, and construction cost are the dependent variables which change according to various independent factors. Therefore, considering the number of dependent variables (more than one), to find the optimal mix design or to rank all mixes, it is necessary to use a multivariate optimization method called the utility function. One of the efficient and suitable methods in this regard is the method proposed by Smith [10].

(1)

$$d_j = \left[\frac{Y_j - \min f_j}{\max f_j - \min f_j} \right]^{t_j}$$

(2)

$$d_j = \left[\frac{\max f_j - Y_j}{\max f_j - \min f_j} \right]^{t_j}$$

(3)

$$D = (d_1 \times d_2 \times d_3 \times \dots \times d_m)^{\frac{1}{m}}$$

In the above equations, d_j represents the utility function value, Y_j is the response, $\max f_j$ is the maximum response value for the j th criterion, $\min f_j$ is the minimum response value for the j th criterion, t_j is the weighting parameter for the j th criterion, D is the objective function, and m is the number of criteria.

The utility function for criteria such as compressive strength, flexural strength, modulus of elasticity, and electrical resistance, where a higher value is desirable, was obtained using Equation (1). For criteria such as chloride ion migration coefficient, volume of permeable pores, and cost, where a lower value is preferred, the utility function was calculated using Equation (2). Then, the overall utility function D as the objective function was calculated according to Equation (3). The utility function value ranges between 0 and 1, where 1 represents the highest desirability and 0 indicates the lowest or no desirability. The weighting parameter t_j reflects the relative importance of each criterion compared to others.

In this study, both technical and economic criteria were considered. Based on interviews and surveys with 10 experts active in the concrete field, the weighting parameter for economic criteria was assigned a coefficient of 1, and for technical criteria—due to their connection with safety and structural stability—a coefficient of 2 was assigned. Therefore, t_j was set to 2 for technical criteria and 1 for economic criteria.

3. Research Findings and Data Analysis

3-1 Technical Evaluation

To produce durable and satisfactory concrete, the selection and application of mixes should be based on combined mechanical properties and durability. In this study, for technical evaluation, three mechanical properties—compressive strength, flexural strength, and modulus of elasticity—and three durability components—accelerated chloride ion migration, volume of permeable voids, and electrical resistance—were all measured at 90 days.

Tables (1) to (6) show the calculated utility coefficients for the six mentioned criteria, each individually based on Equations (1) or (2). Table (7) presents the technical utility coefficient for the mix designs considering all six mechanical and durability factors simultaneously, calculated using Equation (3). Finally, Table (8) ranks the mix designs from 1 to 21.

The results of Table (1) indicate that approximately all mixes containing pumice, zeolite, and slag exhibited better compressive strength performance compared to the control sample. The best mix designs in this regard were, in order: S40, Z40, Z25, and P40. The weakest mixes were L40 and L60. Another noteworthy point is that mixes with 60% replacement (except L60) performed similarly or even better than the control mix. A similar trend is observed for flexural strength and modulus of elasticity, as shown in Tables (2) and (3).

From a durability perspective (results in Tables 4 to 6), the impact of replacement materials was more pronounced than on mechanical properties, as the obtained utility coefficients were closer to 1 or showed significant differences compared to the control sample's utility coefficients. Another important point is the utility coefficient of the control sample for mechanical and durability properties. This coefficient was slightly below average for mechanical properties but was very low for durability, indicating low desirability of the durability component and highlighting the need for special attention to durability issues in this type of cement. The results show that, in terms of durability, the best mix designs are Z40, Z25, P40, and S40.

Table (1): Utility Coefficient of Compressive Strength for Mix Designs (MPa)

Compressive Strength Utility	Maximum Compressive Strength	Minimum Compressive Strength	Compressive Strength	Sample
0.45	46.5	5.5	24.10	C
0.64			31.90	Z5
0.70			34.30	Z15
0.90			42.50	Z25
0.98			45.80	Z40
0.60			30.20	Z60
0.49			25.70	S5
0.64			31.90	S15
0.72			35.00	S25
1.00			46.50	S40
0.40			22.10	S60
0.62			30.90	P5
0.63			31.20	P15
0.71			34.50	P25
0.73			35.30	P40
0.56			28.60	P60
0.34			19.5	L5
0.31			18.4	L15
0.48			25.3	L25
0.15			11.7	L40
0.01	5.5	L60		

Table (2): Utility Coefficient of Flexural Strength of Mix Designs (MPa)

Flexural Strength Utility Coefficient	Minimum Flexural Strength	Minimum Flexural Strength	Flexural Strength	Sample
0.40	2.9	0.75	1.60	C
0.72			2.30	Z5
0.77			2.40	Z15
0.86			2.60	Z25
1.00			2.90	Z40
0.40			1.60	Z60
0.58			2.00	S5
0.63			2.10	S15
0.67			2.20	S25
0.86			2.60	S40
0.49			1.80	S60
0.63			2.10	P5
0.72			2.30	P15
0.77			2.40	P25
0.95			2.80	P40
0.40			1.60	P60
0.35			1.50	L5
0.21			1.20	L15
0.07			0.90	L25
0.02			0.80	L40
0.01	0.75	L60		

Table (3): Utility Coefficient of Elastic Modulus of Mix Designs (MPa)

Elastic Modulus Utility Coefficient	Maximum Elastic Modulus	Minimum Elastic Modulus	Elastic Modulus	Sample Code
0.41	47	8.1	24.00	C
0.72			36.0	Z5
0.74			37.0	Z15
0.85			41.0	Z25
1.00			47.0	Z40
0.47			26.3	Z60
0.89			42.9	S5
0.82			40.1	S15
0.70			35.4	S25
0.90			43.3	S40
0.50			27.4	S60
0.51			28.0	P5
0.61			32	P15
0.72			36	P25
0.90			43	P40
0.46			26	P60
0.36			22.1	L5
0.27			18.6	L15
0.19			15.4	L25
0.11			12.3	L40
0.01	8.1	L60		

Table (4): Utility Coefficient of Accelerated Chloride Ion Migration of Mix Designs ($\times 10^{-12}$ m²/s)

Utility Coefficient of Accelerated Chloride Ion Migration	Minimum Migration Coefficient	Maximum Migration Coefficient	Migration Coefficient	Sample Code
0.53	0.9	12	6.10	C
0.71			4.1	Z5
0.73			3.9	Z15
0.85			2.6	Z25
1.00			0.9	Z40
0.97			1.2	Z60
0.69			4.3	S5
0.80			3.1	S15
0.92			1.8	S25
0.96			1.3	S40
0.58			5.6	S60
0.64			4.9	P5
0.67			4.61	P15
0.68			4.43	P25
0.96			1.32	P40
0.88			2.24	P60
0.51			6.3	L5
0.43			7.2	L15
0.47			6.8	L25
0.45			7.0	L40
0.01			12.0	L60

Table (5): Utility Coefficient of Electrical Resistivity of Mix Designs ($k\Omega \cdot \text{cm}$)

Electrical Resistivity Utility Coefficient	Maximum Electrical Resistivity	Minimum Electrical Resistivity	Electrical Resistivity	Sample Code
0.02	167.2	18.4	21.30	C
0.12			36.6	Z5
0.36			72.6	Z15
0.62			110.3	Z25
1.00			167.2	Z40
0.34			68.3	Z60
0.09			32.0	S5
0.01			18.4	S15
0.07			28.9	S25
0.44			84.0	S40
0.22			50.9	S60
0.02			22.1	P5
0.05			25.6	P15
0.44			83.2	P25
0.59			106.3	P40
0.42			81.3	P60
0.00			18.5	L5
0.03			23.1	L15
0.03			23.6	L25
0.01			20.1	L40
0.01	19.5	L60		

Table (6): Utility Coefficient of Accessible Pore Volume in Mix Design

Permeable Pores Utility Coefficient	Minimum Volume of Permeable Pores (%)	Maximum Volume of Permeable Pores (%)	Volume of Permeable Pores (%)	Sample Code
0.01	6.90	10.5	10.50	C
0.61			8.30	Z5
0.89			7.30	Z15
0.94			7.10	Z25
1.00			6.90	Z40
0.81			7.60	Z60
0.56			8.50	S5
0.86			7.40	S15
0.89			7.30	S25
0.94			7.10	S40
0.75			7.80	S60
0.50			8.70	P5
0.75			7.80	P15
0.81			7.60	P25
0.89			7.30	P40
0.72			7.90	P60
0.67			8.1	L5
0.61			8.3	L15
0.36			9.2	L25
0.17			9.9	L40
0.33	9.3	L60		

Table (7): Technical Utility Coefficient of the Mix Design

Void Volume Utility Coefficient	Chloride Ion Penetration Utility Coefficient	Chloride Ion Penetration Utility Coefficient	Electrical Resistance Utility Coefficient	Elastic Modulus Utility Coefficient	Flexural Strength Utility Coefficient	Compressive Strength Utility Coefficient	Sample Code
0.02	0.01	0.53	0.02	0.41	0.4	0.45	C
0.26	0.61	0.71	0.12	0.72	0.72	0.64	Z5
0.45	0.89	0.73	0.36	0.74	0.77	0.7	Z15
0.69	0.94	0.85	0.62	0.85	0.86	0.9	Z25
0.98	1	1	1	1	1	0.98	Z40
0.31	0.81	0.97	0.34	0.47	0.4	0.6	Z60
0.2	0.56	0.69	0.09	0.89	0.58	0.49	S5
0.13	0.86	0.8	0.01	0.82	0.63	0.64	S15
0.27	0.89	0.92	0.07	0.7	0.67	0.72	S25
0.67	0.94	0.96	0.44	0.9	0.86	1	S40
0.21	0.75	0.58	0.22	0.5	0.49	0.4	S60
0.11	0.5	0.64	0.02	0.51	0.63	0.62	P5
0.19	0.75	0.67	0.05	0.61	0.72	0.63	P15
0.46	0.81	0.68	0.44	0.72	0.77	0.71	P25
0.67	0.89	0.96	0.59	0.9	0.95	0.73	P40
0.27	0.72	0.88	0.42	0.46	0.4	0.56	P60
0.05	0.67	0.51	0.01	0.36	0.35	0.34	L5
0.05	0.61	0.43	0.03	0.27	0.21	0.31	L15
0.03	0.36	0.47	0.03	0.19	0.07	0.48	L25
0.006	0.17	0.45	0.01	0.11	0.02	0.15	L40
0.004	0.33	0.01	0.01	0.01	0.01	0.01	L60

Table (8): Ranking of Mixture Designs Based on Technical Desirability

Sample Rank	Technical Utility Coefficient (Mechanical Properties and Durability)	Sample Code
1	0.98	Z40
2	0.83	Z25
3	0.67	P40
4	0.67	S40
5	0.46	P25
6	0.45	Z15
7	0.31	Z60
8	0.27	P60
9	0.27	S25
10	0.26	Z5
11	0.21	S60
12	0.20	S5
13	0.19	P15
14	0.13	S15
15	0.11	P5
16	0.05	L5
17	0.05	L15
18	0.03	L25
19	0.02	C
20	0.006	L40
21	0.004	L60

3-2. Economic Evaluation

To evaluate the economic aspect of the mixture designs, the cost of each material used in the sample production was first estimated based on the current market prices in the country (at the time of production). Using these estimates, the total cost of materials for one cubic meter of each sample was calculated. Table (9) shows the final material cost of the control sample, and Tables (10) to (13) present sample calculations of the material costs for mixtures Z40, P15, S60, and L25. The material costs for all mixture designs are also illustrated in Figure (1).

Table (9): Calculation of Material Cost per Cubic Meter of the Control Sample

Price of One Cubic Meter of Concrete (Toman)	Total Price of Each Material (Toman)	Unit Price of Material (Toman)	Quantity	Type of Material Used
11,183,960	11,000,000	20,000	550	Cement (kg)
	182,860	120	1524	Sand (kg)
	0	0	0	Substitute materials (kg)
	0	75000	0	Superplasticizer (g)
	1,100	5	220	Water (liters)

Table (10): Calculation of Material Cost per Cubic Meter of Concrete from Mixture Design P15

Price per cubic meter (Toman)	Total price of each material (Toman)	Unit price of materials (Toman)	Quantity	Type of materials used
9,760,387	9,350,000	20,000	468	Cement (kg)
	178,287	120	1486	Sand (kg)
	99,000	1,200	83	Pumice (kg)
	132000	75,000	1.76	Superplasticizer (liters)
	1,100	5	220	Water (liters)

Table (11): Calculation of Material Cost per Cubic Meter of Concrete from Mixture Design Z40

Price of One Cubic Meter of Concrete (Toman)	Total Price of Each Material (Toman)	Unit Price of Material (Toman)	Quantity	Type of materials used
7,577,636	6,600,000	20,000	330	Cement (kg)
	171,036	120	1425	Sand (kg)
	198,000	900	220	Zeolite (kg)
	607,500	75,000	8.1	Superplasticizer (liters)
	1,100	5	220	Water (liters)

Table (12): Calculation of Material Cost per Cubic Meter of Concrete from Mixture Design L25

Price of One Cubic Meter of Concrete (Toman)	Total Price of Each Material (Toman)	Unit Price of Material (Toman)	Quantity	Type of Material Used
8,521,049	8,250,000	20,000	413	Cement (kg)
	177,449	120	1479	Sand (kg)
	75,625	550	138	پودر سنگ (کیلوگرم)
	16,875	75,000	0.2	فوق روانساز (لیتر)
	1,100	5	220	Water (liters)

Table (13): Calculation of Material Cost per Cubic Meter of Concrete from Mixture Design S60

Price of One Cubic Meter of Concrete (Toman)	Total Price of Each Material (Toman)	Unit Price of Material (Toman)	Quantity	Type of Material Used
5,110,062	4,400,000	20,000	220	Cement (kg)
	176,462	120	1471	Sand (kg)
	330,000	1,000	330	Slag (kilograms)
	202,500	75,000	3	Superplasticizer (liters)
	1,100	5	220	Water (liters)

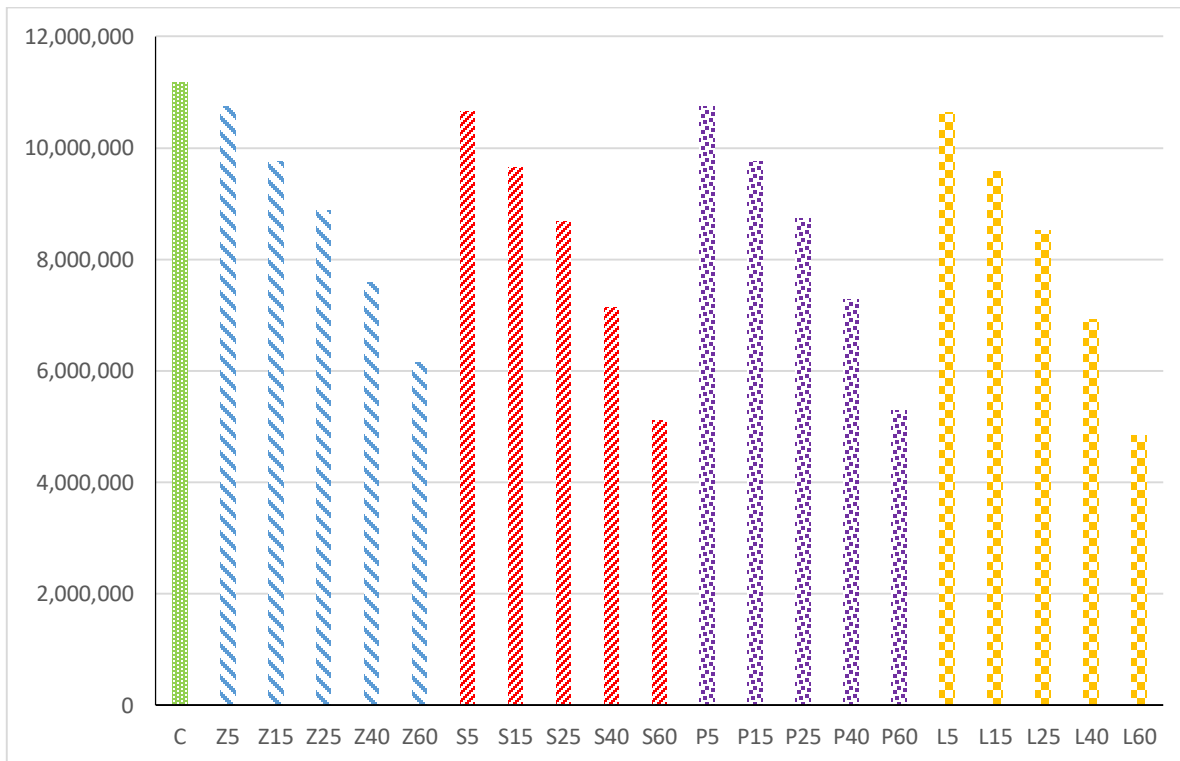


Figure (1): Material Cost per Cubic Meter of the Research Mixtures (in Tomans)

As shown in Figure (1), the highest production cost is related to the control sample, while the lowest cost corresponds to the 60% powder replacement mixture. According to the results presented in Table (14), the control mixture is considered the least desirable — or more accurately, the least economically beneficial — as it has the lowest economic desirability index.

It should be noted that, since cost is a variable where lower values are more desirable, the desirability index has been calculated using Equation (2).

The high cost of the control sample is primarily due to the high price of cement compared to the alternative materials. As previously discussed, calcium aluminate cement is significantly more expensive than Portland cement. Therefore, replacing it with industrial by-products or mineral materials — which are considerably cheaper — has had a substantial effect on reducing the final cost of the product. A comparison of the mixtures made with replacement materials to the control sample shows a cost reduction ranging from approximately **5% to 57%**.

Obviously, cost reduction alone — without considering technical (and even environmental) aspects — is not acceptable. Therefore, the aim of this part of the study is to identify the optimal mixture design by considering all three aspects: **technical, economic, and environmental**.

The results indicate that the mixtures with 60% replacement are clearly much cheaper than other designs. However, since in most technical tests, the Z40, P40, and S40 mixtures demonstrated the best performance, it is logical to focus more on the cost reduction achieved in these three. The cost

reductions due to replacement in these mixtures were **36%**, **35%**, and **32%**, respectively. This level of cost reduction (around one-third), when combined with significant improvements in mechanical properties and durability, can be considered a remarkable achievement.

Table (14): Economic Desirability Index of the Mixture Designs

Economic Utility Coefficient	Minimum	Maximum	Total Cost (Toman per cubic meter)	Sample Code
0.01	4,833,249	11,184,077	11,184,077	C
0.07			10,724,169	Z5
0.23			9,727,660	Z15
0.36			8,883,025	Z25
0.57			7,577,636	Z40
0.79			6,143,367	Z60
0.08			10,658,373	S5
0.24			9,648,396	S15
0.39			8,681,544	S25
0.64			7,139,391	S40
0.96			5,110,062	S60
0.08			10,663,578	P5
0.23			9,722,137	P15
0.39			8,730,071	P25
0.61			7,278,535	P40
0.93			5,298,152	P60
0.08			10,645,774	L5
0.25			9,574,974	L15
0.42			8,521,049	L25
0.67			6,926,099	L40
1.00	4,833,249	L60		

4. Conclusion:

In this study, 21 mixture designs were prepared by partially replacing calcium aluminate cement with various alternative materials at different percentages. The goal was to identify the best mixture designs in terms of mechanical properties, durability, and production cost. Since the number of variables exceeded one item, the desirability function was used for optimization and ranking. The results showed that:

1. The technical desirability index of the mixture designs was obtained considering six factors related to mechanical properties and durability simultaneously. From this perspective, the best mixture designs were Z25, Z40, and P40.
2. The least desirable designs from the technical point of view were L60, L40, and the control sample (C).
3. Economically, based on the final production cost, the most desirable mixture designs were L60, Z40, and S40, respectively.
4. The least desirable designs in terms of cost were S5, Z25, and the control sample (C).
5. This prioritization helps engineers select the appropriate mixture design based on their expectations of calcium aluminate cement and the priority of project criteria.

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